Chapter 4. Determinants

Expansion of Determinants

1 Mark Questions

1. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then write the value of x.

Delhi 2014

Firstly, expand both determinants, which gives equation in x and then solve that equation for finding the value of x.

Given, $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$

$\Rightarrow \quad 2x^2 - 40 = 18 - (-14)$

$\Rightarrow \quad 2x^2 - 40 = 18 + 14$

$\Rightarrow \quad 2x^2 - 40 = 32$

$\Rightarrow \quad 2x^2 = 32 + 40$

$\Rightarrow \quad 2x^2 = 72 \quad \Rightarrow \quad x^2 = 36$

$\therefore \quad x = \pm 6$ **(1)**

2. If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, then find the value of x.

All India 2014

Given, $$\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$$

On expanding the determinant of both sides, we get

$$3x \times 4 - (-2) \times 7 = 8 \times 4 - 6 \times 7$$

$$\Rightarrow \quad 12x - (-14) = 32 - 42$$

$$\Rightarrow \quad 12x + 14 = -10$$

$$\Rightarrow \quad 12x = -10 - 14$$

$$\Rightarrow \quad 12x = -24 \Rightarrow x = -\frac{24}{12}$$

$$\therefore \quad x = -2$$

(1)

3. If A is a 3×3 matrix, $|A| \neq 0$ and $|3A| = k|A|$, then write the value of k.

Foreign 2014

We know that, if A is a square matrix of order n. Then, $|kA| = k^n|A|$.

Here, the matrix A is of order 3.

$$\therefore \quad |3A| = (3)^3|A| = 27|A|$$

On comparing with $k|A|$, we get **(1)**

$$k = 27$$

4. Find (adj A), if $A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$. Delhi 2014C

Given, $A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$

$$\therefore \qquad |A| = \begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix} = 15 - 14 = 1$$

We know that, $|\text{adj}(A)| = |A|^{n-1}$, where n is order of determinant.

$\therefore$ $|\text{adj}(A)| = |1|^{2-1} \Rightarrow |\text{adj}(A)| = 1$ **(1)**

5. Write the value of the determinant $\begin{vmatrix} p & p+1 \\ p-1 & p \end{vmatrix}$. Delhi 2014C

Suppose $A = \begin{vmatrix} p & p+1 \\ p-1 & p \end{vmatrix}$

On expanding, we get

$$A = p^2 - (p-1)(p+1)$$

$$\Rightarrow \qquad A = p^2 - (p^2 - 1^2)$$

$$[\because a^2 - b^2 = (a+b)(a-b)]$$

$$\Rightarrow \qquad A = p^2 - p^2 + 1$$

$$\therefore \qquad A = 1$$

6. If A is a square matrix of order 3 such that $|\text{adj}(A)| = 64$, then find $|A|$. Delhi 2013C

We know that, for a square matrix of order n, $|\text{adj}(A)| = |A|^{n-1}$ have $n = 3$

$\therefore \quad |\text{adj}(A)| = |A|^{3-1} = |A|^2$

Given, $|\text{adj}A| = 64$, $\quad 64 = |A|^2 \Rightarrow (8)^2 = |A|^2$

$\Rightarrow \quad |A| = \pm 8$ [taking square root] **(1)**

7. If $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$, then find the value of x. **Delhi 2013C**

> Expand both determinants which gives equation in x and then solve that equation for finding the value of x.

Given, $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$

$\Rightarrow 2x(x+1) - (x+3)(2x+2) = 3 - 15$

$\Rightarrow 2x^2 + 2x - (2x^2 + 8x + 6) = -12$

$\Rightarrow \quad -6x - 6 = -12$

$\Rightarrow \quad 6x = 6$

$\therefore \quad x = 1$ **(1)**

8. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, then write the value of x. **Delhi 2013**

Given, $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$

$\Rightarrow (x+1)(x+2) - (x-3)(x-1) = 12 + 1$

$\Rightarrow (x^2 + 3x + 2) - (x^2 - 4x + 3) = 13$

$\Rightarrow \quad 7x - 1 = 13$

$\Rightarrow \quad 7x = 14$

$\therefore \quad x = 2$ **(1)**

9. If A_{ij} is the cofactor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then write the value of $a_{32} \cdot A_{32}$. **All India 2013; HOTS**

Let $A = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

Here, $a_{32} = 5$

Given, A_{ij} is the cofactor of the element a_{ij} of A.

Then, $A_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix}$

$= (-1)^5 (8 - 30) = -(-22) = 22$

$\therefore \; a_{32} \cdot A_{32} = 5 \times 22 = 110$ **(1)**

10. Let A be a square matrix of order 3×3. Write the value of $|2A|$, where $|A| = 4$. **All India 2012**

We know that, for a square matrix A of order n,

$$|kA| = k^n \cdot |A|$$

Here, $|2A| = 2^3 \cdot |A|$ $\quad [\because$ order of A is $3 \times 3]$

$= 2^3 \times 4 = 8 \times 4 = 32$ $\quad [\because$ put $|A| = 4]$ **(1)**

11. If $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, then write the minor of the element a_{23}. **Delhi 2012**

Minor of the element

$a_{23} = \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = 10 - 3 = 7$ **(1)**

12. If the determinant of matrix A of order 3×3 is of value 4, then write the value of $|3A|$.

All India 2012C

Given, the order of matrix A is 3×3 and

$$|A| = 4$$

$$\Rightarrow \quad |3A| = 3^3 \cdot |A| \qquad [\because |KA| = K^n \cdot |A|]$$

$$= 3^3 \cdot 4 = 27 \cdot 4 = 108 \qquad \textbf{(1)}$$

13. For what value of x, $A = \begin{bmatrix} 2(x+1) & 2x \\ x & x-2 \end{bmatrix}$ is a singular matrix?

All India 2011C

> For a singular matrix, $|A| = 0$. Use this relation and solve it.

Matrix A is said to be singular, if $|A| = 0$

$$\therefore \quad \begin{vmatrix} 2x+2 & 2x \\ x & x-2 \end{vmatrix} = 0$$

$$\Rightarrow \quad (2x+2)(x-2) - 2x^2 = 0$$

$$\Rightarrow \quad 2x^2 - 2x - 4 - 2x^2 = 0 \Rightarrow -2x = 4$$

$$\therefore \quad x = -2 \qquad \textbf{(1)}$$

14. For what value of x, the matrix $\begin{bmatrix} 2x+4 & 4 \\ x+5 & 3 \end{bmatrix}$ is a singular matrix?

All India 2011C

Let $A = \begin{bmatrix} 2x+4 & 4 \\ x+5 & 3 \end{bmatrix}$

If matrix A is singular, then

$$|A| = 0$$

$$\therefore \quad \begin{vmatrix} 2x+4 & 4 \\ x+5 & 3 \end{vmatrix} = 0$$

$$\Rightarrow \quad (2x+4) \times 3 - (x+5) \times 4 = 0$$

$$\Rightarrow 6x + 12 - 4x - 20 = 0 \quad \Rightarrow \quad 2x = 8$$

$$\therefore \quad x = 4$$ **(1)**

15. For what value of x, the matrix $\begin{bmatrix} 2x & 4 \\ x+2 & 3 \end{bmatrix}$ is a singular matrix? Delhi 2011C

Do same as Que. 14. [**Ans.** $x = 4$]

16. For what value of x, matrix $\begin{bmatrix} 6-x & 4 \\ 3-x & 1 \end{bmatrix}$ is a singular matrix? Delhi 2011C

Do same as Que. 14. [**Ans.** $x = 2$]

17. For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is a singular? Delhi 2011

Do same as Que. 14. [**Ans.** $x = 3$]

18. Evaluate $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$.

All India 2011; HOTS

Firstly, expand the determinant and use the trigonometric relation to calculate the value of determinant.

$$A = \begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$$

On expanding, we get

$$A = (\cos 15^\circ \cos 75^\circ - \sin 15^\circ \sin 75^\circ)$$
$$= \cos(15^\circ + 75^\circ)$$
$$[\because \cos x \cos y - \sin x \sin y = \cos(x + y)]$$
$$= \cos 90^\circ = 0 \qquad [\because \cos 90^\circ = 0] \; \textbf{(1)}$$

19. If $\begin{vmatrix} x & x \\ 1 & x \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$, then write the positive value of x.

Foreign 2011; All India 2008C

Expand both determinants which gives equation in x and then solve that equation for finding the value of x.

Given, $\begin{vmatrix} x & x \\ 1 & x \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$

On expanding, we get

$$x^2 - x = 6 - 4$$
$$\Rightarrow \quad x^2 - x - 2 = 0 \Rightarrow (x - 2)(x + 1) = 0$$
$$\therefore \quad x = 2 \text{ or } -1$$

Hence, the positive value of x is 2. **(1)**

20. What is the value of determinant $\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$?

Delhi 2010

Determinant can be easily expand either corresponding to a row or column which have maximum zeroes.

Given, determinant

$$A = \begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$$

$$\Rightarrow \quad |A| = -2\,(12 - 16)$$

[$\because$ expanding along R_1]

$$= -2\,(-4) = 8$$ **(1)**

21. Find the minor of the element of second row and third column in the determinant

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}.$$

Delhi 2010

Minor of the element of second row and third column is given by

$$M_{23} = \begin{vmatrix} 2 & -3 \\ 1 & 5 \end{vmatrix} = 10 + 3 = 13$$ **(1)**

22. If $A = \begin{bmatrix} 3 & 1 \\ 2 & -3 \end{bmatrix}$, then find $|\text{adj } A|$.

Delhi 2010C; HOTS

Given, $A = \begin{bmatrix} 3 & 1 \\ 2 & -3 \end{bmatrix}$

Cofactors of A are

$$C_{11} = -3, C_{12} = -2, C_{21} = -1, C_{22} = 3$$

We know that, adjoint $A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

$\therefore \quad \text{adj}(A) = \begin{bmatrix} -3 & -2 \\ -1 & 3 \end{bmatrix}^T = \begin{bmatrix} -3 & -1 \\ -2 & 3 \end{bmatrix}$

Now $|\text{adj}(A)| = \begin{vmatrix} -3 & -1 \\ -2 & 3 \end{vmatrix} = -3 \times 3 - (-1 \times -2)$

$$= -9 - 2 = -11$$

$\Rightarrow \quad |\text{adj}(A)| = -11$ **(1)**

Alternate Method

Here, $|A| = \begin{vmatrix} 3 & 1 \\ 2 & -3 \end{vmatrix} = -9 - 2 = -11$

Using the result

$$|\text{adj}(A)| = |A|^{n-1}$$

where, n is order of a determinant, we get

$|\text{adj}(A)| = (-11)^{2-1} = -11$ **(1)**

23. If $|A| = 2$, where A is a 2×2 matrix, then find $|\text{adj } A|$. All India 2010C

Given, $|A| = 2$, where A is a 2×2 matrix.

We know that, $|\text{adj}(A)| = |A|^{n-1}$, where n is the order of matrix. Here, we have

$$n = 2 \text{ and } |A| = 2$$

$\therefore \quad |\text{adj}(A)| = (2)^{2-1}$

$\Rightarrow \quad |\text{adj}(A)| = 2$ **(1)**

24. What positive value of x makes following pair of determinants equal?

$$\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix} = \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$$

All India 2010

Given, $\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix} = \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$

On expanding, we get

$$2x^2 - 15 = 32 - 15 \Rightarrow 2x^2 - 15 = 17$$

$$\Rightarrow \quad 2x^2 = 32 \quad \Rightarrow \quad x^2 = 16$$

$$\Rightarrow \quad x = \pm 4$$ **(1)**

Hence, for $x = 14$, given pair of determinants is equal.

25. If A is a non-singular matrix of order 3 and $|\text{adj } A| = |A|^k$, then what is the value of k?

All India 2009C; HOTS

We know that, for a square matrix of order n

$|\text{adj}(A)| = |A|^{n-1}$

Here, the order of $A = 3 \times 3$, then $n = 3$

$\therefore \quad |\text{adj}(A)| = |A|^2$ ...(i)

But $\quad |\text{adj}(A)| = |A|^k$ [given] ...(ii)

From Eqs. (i) and (ii), we get

$$k = 2$$ **(1)**

26. Evaluate $2\begin{vmatrix} 7 & -2 \\ -10 & 5 \end{vmatrix}$.

Delhi 2009C

$$2\begin{vmatrix} 7 & -2 \\ -10 & 5 \end{vmatrix} = 2\,[35 - (20)] = 2\,(35 - 20)$$

$$= 2 \times 15 = 30$$ **(1)**

NOTE Suppose we want to multiply with 2 inside the determinant, then we do not multiply each element of determinant. Here, we multiply any one row or column by 2.

27. Find x from equation $\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix} = 0.$ **All India 2009**

Given, $\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix} = 0$

$\Rightarrow \quad 2x^2 - 8 = 0 \quad \Rightarrow \quad 2x^2 = 8 \quad \Rightarrow x^2 = 4$

$\therefore \quad x = \pm 2$ **(1)**

28. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then find the value of k, if $|2A| = k \cdot |A|$. **Foreign 2009**

Given, $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$ and $|2A| = k \cdot |A|$

$\Rightarrow \quad 2^2 \cdot |A| = k \cdot |A|$

[$\because$ for a square matrix of order 2 $|kA| = k^2 |A|$, k is any scalar]

$\therefore \quad k = 4$ **(1)**

29. Evaluate $\begin{vmatrix} 2\cos\theta & -2\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$. **Delhi 2008C**

Suppose $A = \begin{vmatrix} 2\cos\theta & -2\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

On expanding, we get

$A = 2\cos^2\theta - (-2\sin^2\theta)$

$= 2\cos^2\theta + 2\sin^2\theta$

$= 2(\cos^2\theta + \sin^2\theta)$

$= 2$ [$\because \cos^2\theta + \sin^2\theta = 1$] **(1)**

30. Evaluate $\begin{vmatrix} a + ib & c + id \\ -c + id & a - ib \end{vmatrix}$. **Delhi 2008; HOTS**

Suppose $A = \begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

On expanding, we get

$$A = (a+ib)(a-ib) - (c+id)(-c+id)$$
$$= (a^2 - i^2b^2) - (-c^2 + i^2d^2)$$
$$[\because (a+b)(a-b) = a^2 - b^2]$$
$$= (a^2 + b^2) - (-c^2 - d^2) \qquad [\because i^2 = -1]$$
$$= a^2 + b^2 + c^2 + d^2 \qquad \textbf{(1)}$$

31. Find for what value of x, is the following matrix singular?

$$\begin{vmatrix} 3-2x & x+1 \\ 2 & 4 \end{vmatrix}$$

Delhi 2008

Do same as Que. 14. **[Ans.** $x = 1$]

32. If $\begin{vmatrix} 2x+5 & 3 \\ 5x+2 & 9 \end{vmatrix} = 0$, then find the value of x.

Foreign 2008

Do same as Que. 27. **[Ans.** $x = -13$]

Properties of Determinants

1 Mark Questions

1. Write the value of $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$. **All India 2014C**

Given, $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$

$$= 2\begin{vmatrix} 8 & 75 \\ 9 & 86 \end{vmatrix} - 7\begin{vmatrix} 3 & 75 \\ 5 & 86 \end{vmatrix} + 65\begin{vmatrix} 3 & 8 \\ 5 & 9 \end{vmatrix}$$

[expanding the determinant along R_1]

$= 2\,(688 - 675) - 7(258 - 375) + 65\,(27 - 40)$

$= 26 + 819 - 845$

$= 845 - 845 = 0$ **(1)**

2. Prove the following, using properties of determinants

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}.$$

$= 2(a+b+c)^3$ **Delhi 2014**

$$\text{LHS} = \begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$$

On applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\text{LHS} = \begin{vmatrix} 2(a+b+c) & a & b \\ 2(a+b+c) & b+c+2a & b \\ 2(a+b+c) & a & c+a+2b \end{vmatrix}$$

On taking $2(a+b+c)$ common from C_1, we get

$$\text{LHS} = 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$

(1/2)

On applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\text{LHS} = 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 0 & b+c+a & 0 \\ 0 & 0 & c+a+b \end{vmatrix}$$

On taking $(a+b+c)$ common from R_2 and R_3, we get

$$\text{LHS} = 2(a+b+c)^3\begin{vmatrix} 1 & a & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

On expanding along R_3, we get

$\text{LHS} = 2(a+b+c)^3\,[(1)\,(1-0)]$ **(1/2)**

$= 2(a+b+c)^3 = \text{RHS}$ **Hence proved.**

3. Using properties of determinants, prove that

$$\begin{vmatrix} x^2+1 & xy & xz \\ xy & y^2+1 & yz \\ xz & yz & z^2+1 \end{vmatrix} = 1 + x^2 + y^2 + z^2.$$

Delhi 2014

$$\text{LHS} = \begin{vmatrix} x^2+1 & xy & xz \\ xy & y^2+1 & yz \\ xz & yz & z^2+1 \end{vmatrix}$$

On taking common factors x, y and z from R_1, R_2 and R_3, we get

$$\text{LHS} = xyz \begin{vmatrix} x+\frac{1}{x} & y & z \\ x & y+\frac{1}{y} & z \\ x & y & z+\frac{1}{z} \end{vmatrix}$$

On applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\text{LHS} = xyz \begin{vmatrix} x+\frac{1}{x} & y & z \\ -\frac{1}{x} & \frac{1}{y} & 0 \\ -\frac{1}{x} & 0 & \frac{1}{z} \end{vmatrix}$$ **(1/2)**

On multiplying and dividing C_1 by x, C_2 by y and C_3 by z and taking common $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ from C_1, C_2 and C_3, we get

$$\text{LHS} = xyz \times \frac{1}{xyz} \begin{vmatrix} x^2+1 & y^2 & z^2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} x^2+1 & y^2 & z^2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

On expanding along R_3, we get

$$\text{LHS} = -1 \times (-z^2) + 1[1(x^2+1) + 1(y^2)]$$

$$= x^2 + y^2 + z^2 + 1 = \text{RHS}$$ **Hence proved.**

(1/2)

4. Write the value of the determinant.

$$\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}.$$

Foreign 2012

$$\text{Let } A = \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

$$\Rightarrow |A| = 6\begin{vmatrix} 17 & 3 & 6 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

[$\because$ taking 6 common from R_1]

$= 6 \times 0 = 0$

[$\because$ two rows (R_1 and R_3) are identical] **(1)**

5. What is the value of $\begin{vmatrix} 4 & a & b+c \\ 4 & b & c+a \\ 4 & c & a+b \end{vmatrix}$?

Foreign 2010

Let $\Delta = \begin{vmatrix} 4 & a & b+c \\ 4 & b & c+a \\ 4 & c & a+b \end{vmatrix}$

On applying $C_2 \to C_2 + C_3$, we get

$$\Delta = \begin{vmatrix} 4 & a+b+c & b+c \\ 4 & a+b+c & c+a \\ 4 & a+b+c & a+b \end{vmatrix}$$

Now, taking common 4 from C_1 and $(a+b+c)$ from C_2, we get

$$\Delta = 4(a+b+c) \begin{vmatrix} 1 & 1 & b+c \\ 1 & 1 & c+a \\ 1 & 1 & a+b \end{vmatrix}$$

$$= 4(a+b+c)(0) = 0 \quad \textbf{(1)}$$

[$\because C_1$ and C_2 are identical]

6. Write the value of $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$.

Delhi 2009

Let $\Delta = \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

On taking $3x$ common from R_3, we get

$$\Delta = 3x \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 2 & 3 & 4 \end{vmatrix}$$

$\Rightarrow \quad \Delta = 0$

[$\because R_1$ and R_3 are identical] **(1)**

7. Write the value of $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$.

All India 2009

Let $\Delta = \begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$

On applying $R_1 \to R_1 + R_2 + R_3$, we get

$$\Delta = \begin{vmatrix} 0 & 0 & 0 \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

$\Rightarrow \quad \Delta = 0$ **(1)**

[$\because$ if a determinant has all elements zero in any of its rows or columns, then value of determinant is zero.]

4 Mark Questions

8. Using properties of determinants, prove that

$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}.$$

All India 2014, 2010C; Delhi 2012, 2010, 2009C

> Firstly, split the determinant along their respective columns and replace determinants, having identical column with zero and arrange the remaining, to get the desired result.

Consider $\Delta = \begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix}$

On splitting Δ along C_1, we get

$$\Delta = \begin{vmatrix} b & c+a & a+b \\ q & r+p & p+q \\ y & z+x & x+y \end{vmatrix}$$

$$+\begin{vmatrix} c & c+a & a+b \\ r & r+p & p+q \\ z & z+x & x+y \end{vmatrix} \quad \textbf{(1)}$$

Again, on splitting both above determinants along their respective second columns, we get

$$\Delta = \begin{vmatrix} b & c & a+b \\ q & r & p+q \\ y & z & x+y \end{vmatrix} + \begin{vmatrix} b & a & a+b \\ q & p & p+q \\ y & x & x+y \end{vmatrix}$$

$$+\begin{vmatrix} c & c & a+b \\ r & r & p+q \\ z & z & x+y \end{vmatrix} + \begin{vmatrix} c & a & a+b \\ r & p & p+q \\ z & x & x+y \end{vmatrix}$$

$$= \begin{vmatrix} b & c & a+b \\ q & r & p+q \\ y & z & x+y \end{vmatrix} + \begin{vmatrix} b & a & a+b \\ q & p & p+q \\ y & x & x+y \end{vmatrix}$$

$$+\begin{vmatrix} c & a & a+b \\ r & p & p+q \\ z & x & x+y \end{vmatrix}$$

$$\left[\because \begin{vmatrix} c & c & a+b \\ r & r & p+q \\ z & z & x+y \end{vmatrix} = 0, \text{ as } C_1 \text{ is identical to } C_2\right]$$

(1)

Similarly, on splitting all above determinants together, we get

$$\Delta = \begin{vmatrix} b & c & a \\ q & r & p \\ y & z & x \end{vmatrix} + \begin{vmatrix} b & c & b \\ q & r & q \\ y & z & y \end{vmatrix} + \begin{vmatrix} b & a & a \\ q & p & p \\ y & x & x \end{vmatrix}$$

$$+\begin{vmatrix} b & a & b \\ q & p & q \\ y & x & y \end{vmatrix} + \begin{vmatrix} c & a & a \\ r & p & p \\ z & x & x \end{vmatrix}$$

$$+\begin{vmatrix} c & a & b \\ r & p & q \\ z & x & y \end{vmatrix}$$

$$\Rightarrow \quad \Delta = \begin{vmatrix} b & c & a \\ q & r & p \\ y & z & x \end{vmatrix} + \begin{vmatrix} c & a & b \\ r & p & q \\ z & x & y \end{vmatrix}$$

[∵ all other determinants have their two columns identical; so their value is 0.]

On applying $C_1 \leftrightarrow C_3$ in first and $C_1 \leftrightarrow C_2$ in second determinant, we get

$$\Delta = -\begin{vmatrix} a & c & b \\ p & r & q \\ x & z & y \end{vmatrix} - \begin{vmatrix} a & c & b \\ p & r & q \\ x & z & y \end{vmatrix}$$

[∵ when any two columns or rows of a determinant are interchanged, its value becomes negative]

On applying $C_2 \leftrightarrow C_3$ in both determinants, we get

$$\Delta = \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} + \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$

$$= 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} \quad \textbf{(1)}$$

Here, we have shown that

$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$

On taking transpose both sides, we get

$$\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$

$$= 2\begin{vmatrix} a & p & x \\ b & q & y \end{vmatrix} \quad \textbf{(1)}$$

$$\begin{vmatrix} c & r & z \end{vmatrix}$$

Hence proved.

NOTE The determinant of matrix A or its transpose A' have same value, i.e. $|A| = |A'|$.

9. Using properties of determinants, prove that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc + bc + ca + ab.$$

All India 2014

Consider LHS $= \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$

On dividing R_1 by a, R_2 by b and R_3 by c and multiplying the determinant by abc, we get

$$\text{LHS} = abc \begin{vmatrix} \frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix} \quad \textbf{(1)}$$

On applying $R_1 \to R_1 + R_2 + R_3$, we get

$$\text{LHS} = abc \begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1 & \frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1 \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & 1 \; {}_{+1} \end{vmatrix} \quad \textbf{(1)}$$

$\frac{}{c}+1$

On taking $\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ common from R_1, we get

$$\text{LHS} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$

(1)

On applying $C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$, we get

$$\text{LHS} = (abc + bc + ca + ab)\begin{vmatrix} 1 & 0 & 0 \\ \frac{1}{b} & 1 & 0 \\ \frac{1}{c} & 0 & 1 \end{vmatrix}$$

$= (abc + bc + ca + ab)\,[1(1-0)]$

[expanding along R_1]

$= abc + bc + ca + ab = \text{RHS}$ **Hence proved.**

(1)

10. Using properties of determinants, prove that

$$\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3.$$

All India 2014, 2009

To prove, $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$

$$\text{LHS} = \begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix}$$

On applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$, we get

$$\text{LHS} = \begin{vmatrix} x+y & x & x \\ 3x+2y & 2x & 0 \\ 7x+5y & 5x & 0 \end{vmatrix}$$ **(1½)**

On expanding along C_3, we get

$$\text{LHS} = x \begin{vmatrix} 3x+2y & 2x \\ 7x+5y & 5x \end{vmatrix}$$ **(1½)**

$$= x\,[5x(3x+2y) - 2x(7x+5y)]$$

$$= x\,[15x^2 + 10xy - (14x^2 + 10xy)] = x^3$$

$$= \text{RHS}$$ **(1)**

Hence proved.

11. Using properties of determinants, prove that

$$\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a+x+y+z).$$

Foreign 2014

Firstly, we make (a+x+y+z) a common factor in any row or column. Now, try to make two zeroes in that row or column and expand the determinant along that row or column.

Given to prove,

$$\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a+x+y+z)$$

$$\text{LHS} = \begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} \quad \textbf{(1)}$$

On applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\text{LHS} = \begin{vmatrix} a+x+y+z & y & z \\ a+x+y+z & a+y & z \\ a+x+y+z & y & a+z \end{vmatrix}$$

$$= (a+x+y+z)\begin{vmatrix} 1 & y & z \\ 1 & a+y & z \\ 1 & y & a+z \end{vmatrix} \quad \textbf{(1)}$$

[taking common $(a+x+y+z)$ from C_1]

On applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\text{LHS} = (a+x+y+z)\begin{vmatrix} 1 & y & z \\ 0 & a & 0 \\ 0 & 0 & a \end{vmatrix} \quad \textbf{(1)}$$

$$= (a+x+y+z)[1(a^2-0)]$$

$$= a^2(a+x+y+z) = \text{RHS} \quad \textbf{(1)}$$

Hence proved.

12. Using properties of determinants, prove that:

$$\begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} = (5x+\lambda)(\lambda-x)^2.$$

Foreign 2014

To prove $\begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix}$

$$= (5x+\lambda)(\lambda-x)^2$$

$$\text{LHS} = \begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix}$$

On applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\text{LHS} = \begin{vmatrix} 5x+\lambda & 2x & 2x \\ 5x+\lambda & x+\lambda & 2x \\ 5x+\lambda & 2x & x+\lambda \end{vmatrix} \quad \textbf{(1)}$$

$$= (5x+\lambda)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x+\lambda & 2x \\ 1 & 2x & x+\lambda \end{vmatrix}$$

[taking $(5x+4)$ common from C_1] **(1)**

On applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\text{LHS} = 5x+\lambda\begin{vmatrix} 1 & 2x & 2x \\ 0 & \lambda-x & 0 \\ 0 & 0 & \lambda-x \end{vmatrix} \quad \textbf{(1)}$$

On expanding along C_1, we get

$$\text{LHS} = (5x+\lambda)[1(\lambda-x)^2 + 0 + 0]$$

$$= (5x+\lambda)(\lambda-x)^2 = \text{RHS} \quad \textbf{(1)}$$

Hence proved.

13. Prove the following, using properties of determinants.

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2.$$

Foreign 2014

To prove, $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$

$$= 4a^2b^2c^2$$

$$\text{LHS} = \begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$$

On taking common a from C_1, b from C_2 and c from C_3, we get

$$\text{LHS} = abc \begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix} \quad \textbf{(1)}$$

On applying $C_1 \to C_1 + C_2 - C_3$ we get

$$\text{LHS} = abc \begin{vmatrix} 0 & c & a+c \\ 2b & b & a \\ 2b & b+c & c \end{vmatrix} \quad \textbf{(1)}$$

Now, applying $R_2 \to R_2 - R_3$, we get

$$\text{LHS} = abc \begin{vmatrix} 0 & c & a+c \\ 0 & -c & a-c \\ 2b & b+c & c \end{vmatrix} \quad \textbf{(1)}$$

On expanding along C_1, we get

LHS $abc\,[2b\,\{c\,(a-c) + c(a+c)\}]$

$$= 2(ab^2c)\,(2ac)$$

$$= 4a^2b^2c^2 = \text{RHS} \quad \textbf{(1)}$$

Hence proved.

NOTE In this type of questions, we only use either row operations or column operations not both at same time.

14. Using properties of determinants, prove that

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(bc+ca+ab).$$

Delhi 2014C, 2011C

To prove, $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix}$ or $\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix}$

$$= (a-b)(b-c)(c-a)(ab+bc+ca)$$

$$\text{LHS} = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix}$$

On multiplying columns C_1, C_2 and C_3 by a, b and c, respectively and dividing the determinant by abc, we get **1**

$$\text{LHS} = \frac{1}{abc}\begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ abc & abc & abc \end{vmatrix} \quad \textbf{(1)}$$

On taking abc common from R_3, we get

$$\text{LHS} = \frac{1}{abc}\cdot abc\begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ 1 & 1 & 1 \end{vmatrix} \quad \textbf{(1/2)}$$

Now, applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$, we get

$$\text{LHS} = \begin{vmatrix} a^2-b^2 & b^2-c^2 & c^2 \\ a^3-b^3 & b^3-c^3 & c^3 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} (a-b)(a+b) & (b-c)(b+c) & c^2 \\ (a-b)(a^2+ab+b^2) & (b-c)(b^2+bc+c^2) & c^3 \\ 0 & 0 & 1 \end{vmatrix}$$

(1)

On taking $(a-b)$ and $(b-c)$ common from C_1 and C_2, respectively and then on expanding along R_3, we get

LHS $= (a-b)(b-c)\cdot 1$

$$\begin{vmatrix} a+b & b+c \\ a^2+ab+b^2 & b^2+bc+c^2 \end{vmatrix}$$ **(1/2)**

On applying $C_2 \to C_2 - C_1$, we get

LHS $= (a-b)(b-c)$

$$\begin{vmatrix} a+b & c-a \\ a^2+ab+b^2 & (c^2-a^2)+b(c-a) \end{vmatrix}$$

On taking $(c-a)$ common from C_2, we get

LHS $= (a-b)(b-c)(c-a)$

$$\begin{vmatrix} a+b & 1 \\ a^2+ab+b^2 & c+a+b \end{vmatrix}$$

$= (a-b)(b-c)(c-a)[(a^2+ab+ac+ab$
$+b^2+bc)-(a^2+ab+b^2)]$

$= (a-b)(b-c)(c-a)(ab+bc+ca)$

$=$ RHS **(1)**

Hence proved.

15. Using properties of determinants, prove that

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2.$$

Delhi 2014C, 2013; Foreign 2009

Firstly, apply $C_1 \to C_1 + C_2 + C_3$ and then take a term common from C_1 and then solve it.

To prove, $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$

LHS $= \begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$

On applying $C_1 \to C_1 + C_2 + C_3$, we get

On applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\text{LHS} = \begin{vmatrix} 1+x+x^2 & x & x^2 \\ 1+x+x^2 & 1 & x \\ 1+x+x^2 & x^2 & 1 \end{vmatrix} \quad \textbf{(1/2)}$$

On taking common $(1+x+x^2)$ from C_1, we get

$$\text{LHS} = (1+x+x^2)\begin{vmatrix} 1 & x & x^2 \\ 1 & 1 & x \\ 1 & x^2 & 1 \end{vmatrix} \quad \textbf{(1/2)}$$

On applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\text{LHS} = (1+x+x^2)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1-x & x(1-x) \\ 0 & x(x-1) & 1-x^2 \end{vmatrix} \quad \textbf{(1)}$$

On expanding along C_1, we get

$$\text{LHS} = (1+x+x^2)\begin{vmatrix} 1-x & x(1-x) \\ -x(1-x) & (1-x^2) \end{vmatrix}$$

$$= (1+x+x^2)\begin{vmatrix} 1-x & x(1-x) \\ -x(1-x) & (1-x)(1+x) \end{vmatrix} \quad \textbf{(1/2)}$$

On taking common $(1-x)$ from C_1 and C_2, we get

$$\text{LHS} = (1+x+x^2)(1-x)^2\begin{vmatrix} 1 & x \\ -x & 1+x \end{vmatrix} \quad \textbf{(1/2)}$$

$$= (1+x+x^2)(1-x)^2(1+x+x^2)$$

$$= \{(1+x+x^2)(1-x)\}^2$$

$$= (1-x^3)^2 = \text{RHS}$$

$$[\because (a^2+b^2+ab)(a-b) = a^3 - b^3] \quad \textbf{(1)}$$

Hence proved.

16. Show that $\Delta = \Delta_1$, where

$$\Delta = \begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix}, \Delta_1 = \begin{vmatrix} A & B & C \\ x & y & z \\ zy & zx & xy \end{vmatrix},$$

All India 2014C

Given, $\Delta = \begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix}$

On taking common x, y and z from R_1, R_2 and R_3 respectively, we get

$$\Delta = xyz \begin{vmatrix} A & x & 1/x \\ B & y & 1/y \\ C & z & 1/z \end{vmatrix}$$

Now, on applying $C_3 \to xyzC_3$, we get

$$\Delta = \frac{xyz}{xyz} \begin{vmatrix} A & x & yz \\ B & y & zx \\ C & z & xy \end{vmatrix} = \begin{vmatrix} A & x & yz \\ B & Y & zx \\ C & z & xy \end{vmatrix}$$

Also, given $\Delta_1 = \begin{vmatrix} A & B & C \\ x & y & z \\ zy & zx & xy \end{vmatrix}$ **(2)**

$$\Delta_1' = \begin{vmatrix} A & x & yz \\ B & y & zx \\ C & z & xy \end{vmatrix}$$

$\therefore$ $\Delta = \Delta_1' \Rightarrow \Delta = \Delta_1$ $[\because |A'| = |A|]$

(2)

17. Using properties of determinants, prove that

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc.$$

All India 2014C, All India 2012

> Firstly, apply $R_1 \to R_1 + R_2 + R_3$ and then take a term common from R_1 and solve it.

To prove, $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$

$$\text{LHS} = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

On applying $R_1 \to R_1 + R_2 + R_3$, we get

$$\text{LHS} = \begin{vmatrix} 2b+2c & 2a+2c & 2a+2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix} \quad \textbf{(1)}$$

$$= 2\begin{vmatrix} b+c & a+c & a+b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

[taking 2 common from R_1]

On applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$, we get

$$\text{LHS} = 2\begin{vmatrix} c & 0 & a \\ b-c & a & -a \\ c & c & a+b \end{vmatrix} \quad \textbf{(1)}$$

On expanding along R_1, we get

$\text{LHS} = 2\,[c\,(a^2 + ab + ac) + a\,(cb - c^2 - ac)]$

$= 2\,[ca^2 + abc + ac^2 + acb - ac^2 - a^2c]$ **(1)**

$= 2\,[2abc] = 4abc = \text{RHS}$ **(1)**

Hence proved.

18. Using properties of determinants, prove that

$$\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c).$$

Delhi 2013C, 2009C

Here, use row operations (or column operations) to make some factors common in one row or column. Then, take that factor outside the determinant and then expand the determinant.

Given, to prove $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix}$

$= (a-b)(b-c)(c-a)(a+b+c)$

$$\text{LHS} = \begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix}$$

On applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\text{LHS} = \begin{vmatrix} 1 & a & a^3 \\ 0 & b-a & b^3 - a^3 \\ 0 & c-a & c^3 - a^3 \end{vmatrix} \quad \textbf{(1)}$$

$$= \begin{vmatrix} 1 & a & a^3 \\ 0 & b-a & (b-a)(b^2+a^2+ab) \\ 0 & c-a & (c-a)(c^2+a^2+ac) \end{vmatrix}$$

$[\because x^3 - y^3 = (x-y)(x^2+y^2+xy)]$

On taking $(b-a)$ and $(c-a)$ common from R_2 and R_3, respectfully, we get

$$\text{LHS} = (b-a)(c-a)\begin{vmatrix} 1 & a & a^3 \\ 0 & 1 & b^2+a^2+ab \\ 0 & 1 & c^2+a^2+ac \end{vmatrix} \quad \textbf{(1)}$$

$= (b-a)(c-a)[1\{c^2+a^2+ac$

$-(b^2+a^2+ab)\}]$

[expanding along C_1]

$= (b-a)(c-a)[c^2-b^2+ac-ab]$ **(1)**

$= (b-a)(c-a)[(c-b)(c+b)+a(c-b)]$

$= (b - a)(c - a)(c - b)(c + b + a)$

$= (a - b)(b - c)(c - a)(a + b + c)$ **(1)**

$= \text{RHS}$ **Hence proved.**

Alternate Method

$$\text{LHS} = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} \qquad [\because |A'| = |A|]$$

On applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$, we get

$$\text{LHS} = \begin{vmatrix} 0 & 0 & 1 \\ a - b & b - c & c \\ a^3 - b^3 & b^3 - c^3 & c^3 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ a - b & b - c & c \\ (a - b)(a^2 + ab + b^2) & (b - c)(b^2 + bc + c^2) & c^3 \end{vmatrix}$$

(1½)

$[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$

On taking $(a - b)$ common from C_1 and $(b - c)$ from C_2, we get

$\text{LHS} = (a - b)(b - c)$

$$\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a^2 + ab + b^2 & b^2 + bc + c^2 & c^3 \end{vmatrix}$$ **(1/2)**

On applying $C_1 \to C_1 - C_2$, we get

$\text{LHS} = (a - b)(b - c)$

$$\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ (a^2 - c^2) + (ab - bc) & b^2 + bc + c^2 & c^3 \end{vmatrix}$$

$$= (a - b)(b - c) \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ (a - c)(a + b + c) & b^2 + bc + c^2 & c^3 \end{vmatrix}$$

$$\left[\begin{array}{l}\because\ (a^2-c^2)+ab-bc \\ =(a-c)(a+c)+b(a-c) \\ =(a-c)(a+b+c)\end{array}\right]$$

On taking $(c-a)(a+b+c)$ common from C_1, we get

$$\text{LHS}=(a-b)(b-c)(c-a)(a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ -1 & b^2+bc+c^2 & c^3 \end{vmatrix}$$ **(1)**

On expanding along column C_1, we get

$$=(a-b)(b-c)(c-a)(a+b+c)\left(-1\times\begin{vmatrix} 0 & 1 \\ 1 & c\end{vmatrix}\right)$$

$$=(a-b)(b-c)(c-a)(a+b+c)[-1(-1)]$$

$$=(a-b)(b-c)(c-a)(a+b+c)$$ **(1)**

$=$ RHS **Hence proved.**

NOTE In this type of questions, we only use either row operations or column operations not both at same time.

19. Using properties of determinants, prove that

$$\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix}$$

$=3(x+y+z)(xy+yz+zx)$. **All India 2013**

To prove, $\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix}$

$= 3(x+y+z)(xy+yz+zx)$

$$\text{LHS} = \begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix}$$

On applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\text{LHS} = \begin{vmatrix} x+y+z & -x+y & -x+z \\ x+y+z & 3y & z-y \\ x+y+z & y-z & 3z \end{vmatrix} \quad \textbf{(1)}$$

On taking common $(x+y+z)$ from C_1, we get

$$\text{LHS} = (x+y+z)\begin{vmatrix} 1 & -x+y & -x+y \\ 1 & 3y & z-y \\ 1 & y-z & 3z \end{vmatrix} \quad \textbf{(1)}$$

On applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$, we get

$$\text{LHS} = (x+y+z)\begin{vmatrix} 1 & -x+y & -x+z \\ 0 & 2y+x & x-y \\ 0 & x-z & 2z+x \end{vmatrix} \quad \textbf{(1)}$$

Now, on expanding along C_1, we get

$$\text{LHS} = (x+y+z)\cdot 1 \cdot \{(2y+x)(2z+x) - (x-y)(x-z)\}$$

$$= (x+y+z)\{4yz + 2xz + 2yx + x^2 - x^2 + xy + zx - yz\}$$

$$= (x+y+z)\cdot(3xy + 3yz + 3zx)$$

$$= 3(x+y+z)\cdot(xy+yz+zx) = \text{RHS} \quad \textbf{(1)}$$

Hence proved.

20. Using properties of determinants, prove that

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^2(x+y).$$

All India 2013

To prove, $\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$

$= 9y^2(x+y)$

$$\text{LHS} = \begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$$

On applying $R_1 \to R_1 + R_2 + R_3$, we get

$$\text{LHS} = \begin{vmatrix} 3x+3y & 3x+3y & 3x+3y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} \quad \textbf{(1)}$$

On taking $(3x + 3y)$ common from R_1, we get

$$\text{LHS} = (3x+3y)\begin{vmatrix} 1 & 1 & 1 \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} \quad \textbf{(1)}$$

On applying $C_2 \to C_2 - C_1$ and
$C_3 \to C_3 - C_1$, we get

$$\text{LHS} = 3(x+y)\begin{vmatrix} 1 & 0 & 0 \\ x+2y & -2y & -y \\ x+y & y & -y \end{vmatrix} \quad \textbf{(1)}$$

Now, expanding along R_1, we get

$$\text{LHS} = 3(x+y)\cdot 1 \cdot [(-2y)\cdot(-y) - (y)\cdot(-y)]$$

$$= 3(x+y)[2y^2 + y^2] = 3(x+y)(3y^2) \quad \textbf{(1)}$$

$$= 9y^2(x+y) = \text{RHS}$$ **Hence proved.**

21. Using properties of determinants, prove that

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix}$$

$$= (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma).$$

Delhi 2012C, 2010C, 2008C

To prove, $\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix}$

$$= (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$$

$$\text{LHS} = \begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix}$$

On applying $R_3 \to R_3 + R_1$, we get

$$\text{LHS} = \begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \alpha+\beta+\gamma & \alpha+\beta+\gamma & \alpha+\beta+\gamma \end{vmatrix}$$

$$= (\alpha+\beta+\gamma) \begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ 1 & 1 & 1 \end{vmatrix}$$ **(1½)**

On applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$, we get

$$\text{LHS} = (\alpha+\beta+\gamma) \begin{vmatrix} \alpha-\beta & \beta-\gamma & \gamma \\ \alpha^2-\beta^2 & \beta^2-\gamma^2 & \gamma^2 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= (\alpha+\beta+\gamma) \begin{vmatrix} \alpha-\beta & \beta-\gamma & \gamma \\ (\alpha-\beta)(\alpha+\beta) & (\beta-\gamma)(\beta+\gamma) & \gamma^2 \\ 0 & 0 & 1 \end{vmatrix}$$

On taking $(\alpha-\beta)$ common from C_1 and $(\beta-\alpha)$ common from C_2, we get

$$\text{LHS} = (\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma) \begin{vmatrix} 1 & 1 & \gamma \\ \alpha+\beta & \beta+\gamma & \gamma^2 \\ 0 & 0 & 1 \end{vmatrix}$$ (1½)

On expanding along R_3, we get

$$\text{LHS} = (\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma) \begin{vmatrix} 1 & 1 \\ \alpha+\beta & \beta+\gamma \end{vmatrix}$$

$$= (\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)(\beta+\gamma-\alpha-\beta)$$

$$= (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$$ **(1)**

$= \text{RHS}$ **Hence proved.**

22. Using properties of determinants, prove that

$$\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix}$$

$$=(a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2).$$

All India 2012C

To prove, $\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix}$

$$= (a-b)(b-c)(c-a)$$
$$(a+b+c)(a^2+b^2+c^2)$$

$$\text{LHS} = \begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix}$$

On applying $C_2 \rightarrow C_2 - C_1$, we get

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$$\text{LHS} = \begin{vmatrix} a^2 & -(b-c)^2 & bc \\ b^2 & -(c-a)^2 & ca \\ c^2 & -(a-b)^2 & ab \end{vmatrix}$$

$$= -\begin{vmatrix} a^2 & b^2+c^2-2bc & bc \\ b^2 & c^2+a^2-2ac & ca \\ c^2 & a^2+b^2-2ab & ab \end{vmatrix} \quad \textbf{(1)}$$

[taking ' − ' common from C_1]

On applying $C_2 \rightarrow C_2 + C_1 + 2C_3$, we get

$$\text{LHS} = -\begin{vmatrix} a^2 & a^2+b^2+c^2 & bc \\ b^2 & a^2+b^2+c^2 & ca \\ c^2 & a^2+b^2+c^2 & ab \end{vmatrix}$$

$$= -(a^2+b^2+c^2)\begin{vmatrix} a^2 & 1 & bc \\ b^2 & 1 & ca \\ c^2 & 1 & ab \end{vmatrix}$$

[taking $(a^2+b^2+c^2)$ common from C_2] **(1)**

On applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$, we get

$$\text{LHS} = -(a^2 + b^2 + c^2) \begin{vmatrix} a^2 - b^2 & 0 & c(b-a) \\ b^2 - c^2 & 0 & a(c-b) \\ c^2 & 1 & ab \end{vmatrix}$$

$$= -(a^2 + b^2 + c^2)(a-b)(b-c) \begin{vmatrix} a+b & 0 & -c \\ b+c & 0 & -a \\ c^2 & 1 & ab \end{vmatrix} \quad \textbf{(1)}$$

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[taking $(a-b)$ common from C_1 and $(b-c)$ from C_3]

On applying $C_1 \to C_1 - C_3$, we get

$$\text{LHS} = -(a^2 + b^2 + c^2)(a-b)(b-c) \begin{vmatrix} a+b+c & 0 & -c \\ a+b+c & 0 & -a \\ c^2 - ab & 1 & ab \end{vmatrix}$$

On expanding along C_2, we get

$$\text{LHS} = -(a^2 + b^2 + c^2)(a-b)(b-c)(-1)^{3+2}(a+b+c)(-a+c)$$

$$= (a-b)(b-c)(c-a)(a+b+c)(a^2 + b^2 + c^2) \quad \textbf{(1)}$$

$= \text{RHS}$ **Hence proved.**

23. Using properties of determinants, prove that

$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2.$$

Delhi 2011; All India 2011C

To prove, $\begin{vmatrix} -a & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$

$$\text{LHS} = \begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$$

On taking a, b and c common from R_1, R_2 and R_3 respectively, we get

$$\text{LHS} = abc \begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} \quad \textbf{(1)}$$

Again, on taking a, b and c common from C_1, C_2 and C_3 respectively, we get

$$\text{LHS} = a^2b^2c^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} \quad \textbf{(1)}$$

On applying $C_1 \to C_1 + C_2$, we get

$$\text{LHS} = a^2b^2c^2 \begin{vmatrix} 0 & 1 & 1 \\ 0 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix} \quad \textbf{(1)}$$

$= a^2b^2c^2\, 2(1+1)$ [expanding along C_1]

$= 4a^2b^2c^2 = \text{RHS}$ **(1)**

Hence proved.

24. Using properties of determinants, prove that

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x).$$

Delhi 2011, 2010C

To prove,

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz\,(x-y)\,(y-z)\,(z-x)$$

$$\text{LHS} = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$$

(1/2)

[taking x, y and z common from C_1, C_2 and C_3, respectively]

On applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$, we get

$$\text{LHS} = xyz \begin{vmatrix} 0 & 0 & 1 \\ x-y & y-z & z \\ x^2-y^2 & y^2-z^2 & z^2 \end{vmatrix}$$ **(1½)**

On expanding along R_1, we get

$$\text{LHS} = xyz \begin{vmatrix} x-y & y-z \\ x^2-y^2 & y^2-z^2 \end{vmatrix}$$ **(1)**

On taking common $(x-y)$ from C_1 and $(y-z)$ from C_2, we get

$$\text{LHS} = xyz\,(x-y)\,(y-z) \begin{vmatrix} 1 & 1 \\ x+y & y+z \end{vmatrix}$$

$= xyz\,(x-y)\,(y-z)\,(z-x)$ **(1)**

$=$ RHS **Hence proved.**

25. Using properties of determinants, prove that

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)\,(4-x)^2.$$

Delhi 2011, 2009

To prove,

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x^2)$$

$$\text{LHS} = \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

On applying $R_1 \to R_1 + R_2 + R_3$, we get

LHS

$$= \begin{vmatrix} x+4+2x+2x & 2x+x+4+2x & 2x+2x+x+4 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

$$= \begin{vmatrix} 5x+4 & 5x+4 & 5x+4 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} \quad \textbf{(1)}$$

$$= (5x+4)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} \quad \textbf{(1/2)}$$

[taking $5x + 4$ common from R_1]

Now, on applying $C_2 \to C_2 - C_1$, we get

$$= (5x+4)\begin{vmatrix} 1 & 0 & 1 \\ 2x & 4-x & 2x \\ 2x & 0 & 4+x \end{vmatrix} \quad \textbf{(1)}$$

On expanding along C_2, we get

$$= (5x+4)(4-x)\begin{vmatrix} 1 & 1 \\ 2x & 4+x \end{vmatrix} \quad \textbf{(1)}$$

$$= (5x+4)(4-x)(4+x-2x)$$

$$= (5x+4)(4-x)^2 = \text{RHS} \quad \textbf{(1/2)}$$

Hence proved.

26. Using properties of determinants, solve the following for x.

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

All India 2011; HOTS

> Firstly, apply some operations and use properties, so that when we expand the determinant, it is easy to simplify.

Given determinant

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

On applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$, we get

$$\begin{vmatrix} 2 & 6 & 12 \\ 4 & 18 & 48 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0 \quad \textbf{(1½)}$$

On taking common 2 from R_1 and R_2, we get

$$2 \times 2 \begin{vmatrix} 1 & 3 & 6 \\ 2 & 9 & 24 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0 \quad \textbf{(1/2)}$$

On applying $R_2 \to R_2 - 2R_1$, we get

$$4 \begin{vmatrix} 1 & 3 & 6 \\ 0 & 3 & 12 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0 \quad \textbf{(1)}$$

On expanding along C_1, we get

$$4[3(3x-64) - 12(2x-27) + (x-8)(3 \times 12 - 3 \times 6)] = 0$$

$$\Rightarrow 4[9x - 192 - 24x + 324 + 18(x-8)] = 0$$

$$\Rightarrow 4[3x - 12] = 0$$

$$\Rightarrow 3x = 12$$

$$\therefore x = 4 \quad \textbf{(1)}$$

27. Using properties of determinants, solve the following for x.

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

All India 2011

The given determinant equation is

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

On applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} 3a-x & a-x & a-x \\ 3a-x & a+x & a-x \\ 3a-x & a-x & a+x \end{vmatrix} = 0 \quad \textbf{(1)}$$

On taking $(3a - x)$ common from C_1, we get

$$(3a-x)\begin{vmatrix} 1 & a-x & a-x \\ 1 & a+x & a-x \\ 1 & a-x & a+x \end{vmatrix} = 0 \quad \textbf{(1/2)}$$

On applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$(3a-x)\begin{vmatrix} 1 & a-x & a-x \\ 0 & 2x & 0 \\ 0 & 0 & 2x \end{vmatrix} = 0 \quad \textbf{(1½)}$$

On expanding along C_1, we get

$$(3a-x)\cdot 1 \cdot \begin{vmatrix} 2x & 0 \\ 0 & 2x \end{vmatrix} = 0$$

$\Rightarrow \quad (3a - x) \cdot 2x \cdot 2x = 0$

$\Rightarrow \quad 4x^2 (3a - x) = 0$

$\therefore \quad x = 0, 3a$ **(1)**

28. Using properties of determinants, solve the following for x.

$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

All India 2011

The given determinant is

$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

On applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} 3x+a & x & x \\ 3x+a & x+a & x \\ 3x+a & x & x+a \end{vmatrix} = 0 \quad \textbf{(1)}$$

On taking common $(3x + a)$ from C_1, we get

$$(3x+a)\begin{vmatrix} 1 & x & x \\ 1 & x+a & x \\ 1 & x & x+a \end{vmatrix} = 0 \quad \textbf{(1/2)}$$

Now, on applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$(3x+a)\begin{vmatrix} 1 & x & x \\ 0 & a & 0 \\ 0 & 0 & a \end{vmatrix} = 0 \quad \textbf{(1½)}$$

On expanding along C_1, we get

$$(3x + a)(1 \cdot a \cdot a) = 0$$

$$\Rightarrow \quad a^2 (3x + a) = 0$$

$$\therefore \quad x = -\frac{a}{3} \quad \textbf{(1)}$$

29. Prove, using properties of determinants

$$\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k).$$

Foreign 2011

Firstly, apply $R_1 \to R_1 + R_2 + R_3$ and then to take common from R_1 and then solve it.

To prove, $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$

$$\text{LHS} = \begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$$

On applying $R_1 \to R_1 + R_2 + R_3$, we get

$$\text{LHS} = \begin{vmatrix} 3y+k & 3y+k & 3y+k \\ y & y+k & y \\ y & y & y+k \end{vmatrix} \quad \textbf{(1)}$$

On taking $(3y+k)$ common from R_1, we get

$$\text{LHS} = (3y+k)\begin{vmatrix} 1 & 1 & 1 \\ y & y+k & y \\ y & y & y+k \end{vmatrix} \quad \textbf{(1/2)}$$

Now, on applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$, we get

$$\text{LHS} = (3y+k)\begin{vmatrix} 0 & 0 & 1 \\ -k & k & y \\ 0 & -k & y+k \end{vmatrix} \quad \textbf{(1½)}$$

On expanding along R_1, we get

$\text{LHS} = (3y+k)\,[1(k^2)]$

$= k^2(3y+k) = \text{RHS}$ **(1)**

Hence proved.

30. Prove, using properties of determinants

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3.$$

Foreign 2011

To prove,

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c)^3$$

$$\text{LHS} = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

On applying $R_1 \to R_1 + R_2 + R_3$, we get

$$\text{LHS} = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

(1)

On taking $(a+b+c)$ common from R_1, we get

$$\text{LHS} = (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

(1/2)

Now, on applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$, we get

$$= (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ b+c+a & -(a+b+c) & 2b \\ 0 & c+a+b & c-a-b \end{vmatrix}$$

(1½)

On taking $(a+b+c)$ common from C_1 and C_2, both, we get

$$\text{LHS} = (a+b+c)^3 \begin{vmatrix} 0 & 0 & 1 \\ 1 & -1 & 2b \\ 0 & 1 & c-a-b \end{vmatrix}$$

On expanding along R_1, we get

$$\text{LHS} = (a+b+c)^3 [1(1)]$$

$$= (a+b+c)^3 = \text{RHS}$$ **(1)**

Hence proved.

31. Prove, using properties of determinants

$$\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3.$$

Foreign 2011; All India 2009C, 2008

To prove, $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix}$

$= 2(x+y+z)^3$

$$\text{LHS} = \begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix}$$

On applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\text{LHS} = \begin{vmatrix} 2x+2y+2z & x & y \\ 2x+2y+2z & y+z+2x & y \\ 2x+2y+2z & x & z+x+2y \end{vmatrix}$$

(1)

On taking $2(x+y+z)$ common from C_1, we get

$$\text{LHS} = 2(x+y+z)\begin{vmatrix} 1 & x & y \\ 1 & y+z+2x & y \\ 1 & x & z+x+2y \end{vmatrix}$$ **(1/2)**

On applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\text{LHS} = 2(x+y+z)\begin{vmatrix} 1 & x & y \\ 0 & y+z+x & 0 \\ 0 & 0 & z+x+y \end{vmatrix}$$

(1½)

On taking common $(x + y + z)$ from R_2 and R_3, both, we get

$$\text{LHS} = 2(x+y+z)^3 \begin{vmatrix} 1 & x & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

On expanding along C_1, we get

$$\text{LHS} = 2(x+y+z)^3 [1(1-0) - 0 + 0]$$

$$= 2(x+y+z)^3 = \text{RHS} \quad \textbf{(1)}$$

Hence proved.

32. Prove, using properties of determinants

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2.$$

All India 2011C; Foreign 2009

To prove,

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

$$\text{LHS} = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$$

On applying $R_1 \to \frac{1}{a}R_1, R_2 \to \frac{1}{b}R_2$ and $R_3 \to \frac{1}{c}R_3$, we get

$$\text{LHS} = abc \begin{vmatrix} a+\frac{1}{a} & b & c \\ a & b+\frac{1}{b} & c \\ a & b & c+\frac{1}{c} \end{vmatrix} \quad \textbf{(1)}$$

On applying $C_1 \to aC_1, C_2 \to bC_2$ and $C_3 \to cC_3$, we get

$$\text{LHS} = \frac{abc}{abc} \begin{vmatrix} a^2+1 & b^2 & c^2 \\ a^2 & b^2+1 & c^2 \\ a^2 & b^2 & c^2+1 \end{vmatrix} \quad \textbf{(1)}$$

On applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\text{LHS} = \begin{vmatrix} 1+a^2+b^2+c^2 & b^2 & c^2 \\ 1+a^2+b^2+c^2 & b^2+1 & c^2 \\ 1+a^2+b^2+c^2 & b^2 & c^2+1 \end{vmatrix}$$

$$= (1+a^2+b^2+c^2) \begin{vmatrix} 1 & b^2 & c^2 \\ 1 & b^2+1 & c^2 \\ 1 & b^2 & c^2+1 \end{vmatrix} \quad \textbf{(1)}$$

[taking $(1+a^2+b^2+c^2)$ common from C_1]

On applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\text{LHS} = (1 + a^2 + b^2 + c^2)\begin{vmatrix} 1 & b^2 & c^2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$ **(1/2)**

On expanding along C_1, we get

$$\text{LHS} = (1 + a^2 + b^2 + c^2)\,[1(1-0)]$$

$$= 1 + a^2 + b^2 + c^2$$ **(1/2)**

$$= \text{RHS}$$ **Hence proved.**

33. Prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3.$$

Delhi 2010; All India 2010C

To prove, $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$

$= 2abc\,(a+b+c)^3$

$$\text{LHS} = \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (a+c)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$$

On applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we get

$$\text{LHS} = \begin{vmatrix} (b+c)^2 & a^2-(b+c)^2 & a^2-(b+c)^2 \\ b^2 & (a+c)^2-b^2 & 0 \\ c^2 & 0 & (a+b)^2-c^2 \end{vmatrix}$$

$$= \begin{vmatrix} (b+c)^2 & (a+b+c)(a-b-c) & (a+b+c)(a-b-c) \\ b^2 & (a+c+b)(a+c-b) & 0 \\ c^2 & 0 & (a+b+c)(a+b-c) \end{vmatrix}$$ **(1½)**

$[\because x^2 - y^2 = (x-y)(x+y)]$

On taking $(a+b+c)$ common from C_2 and C_3, we get

LHS $= (a+b+c)^2$

$$\begin{vmatrix} (b+c)^2 & a-b-c & a-b-c \\ b^2 & a+c-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

On applying $R_1 \rightarrow R_1 - (R_2 + R_3)$, we get

LHS $= (a+b+c)^2$

$$\begin{vmatrix} 2bc & -2c & -2b \\ b^2 & a+c-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$ **(1/2)**

On applying $C_2 \rightarrow C_2 + \frac{1}{b}C_1$,

$C_3 \rightarrow C_3 + \frac{1}{c}C_1$, we get

$$\text{LHS} = (a+b+c)^2 \begin{vmatrix} 2bc & 0 & 0 \\ b^2 & a+c & b^2/c \\ c^2 & c^2/b & a+b \end{vmatrix}$$ **(1)**

On expanding along R_1, we get

LHS $= (a+b+c)^2 [2bc(a^2 + ab$

$+ ac + bc - bc)]$

$= (a+b+c)^2 [2bc\,(a^2 + ab + ac)]$

$= (a+b+c)^2 \cdot 2abc\,(a+b+c)$

$= 2abc\,(a+b+c)^3 = \text{RHS}$ **(1)**

Hence proved.

34. Using properties of determinants, prove that

$$\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = xyz + xy + yz + zx.$$

All India 2009, 2008

To prove, $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$

$= xyz + xy + yz + zx$

$$\text{LHS} = \begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$$

On dividing R_1 by x, R_2 by y and R_3 by z and multiplying the determinant by xyz, we get

$$\text{LHS} = xyz \begin{vmatrix} \frac{1}{x}+1 & \frac{1}{x} & \frac{1}{x} \\ \frac{1}{y} & \frac{1}{y}+1 & \frac{1}{y} \\ \frac{1}{z} & \frac{1}{z} & \frac{1}{z}+1 \end{vmatrix}$$ **(1)**

On applying $R_1 \to R_1 + R_2 + R_3$, we get

$$= xyz \begin{vmatrix} 1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z} & 1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z} & 1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z} \\ \frac{1}{y} & \frac{1}{y}+1 & \frac{1}{y} \\ \frac{1}{z} & \frac{1}{z} & \frac{1}{z}+1 \end{vmatrix}$$

(1)

On taking $\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ common from R_1,

we get LHS $= xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1/y & (1/y)+1 & 1/y \\ 1/z & 1/z & (1/z)+1 \end{vmatrix}$$ **(1/2)**

On applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we get

$$\text{LHS} = (xyz + yz + zx + xy)\begin{vmatrix} 1 & 0 & 0 \\ 1/y & 1 & 0 \\ 1/z & 0 & 1 \end{vmatrix}$$ **(1)**

$= (xyz + xy + yz + zx)\,[1(1-0)]$

[$\because$ expanding along R_1]

$= xyz + xy + yz + zx = \text{RHS}$ **(1/2)**

Hence proved.

35. Prove that $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1.$

All India 2009

To prove, $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1$

$$\text{LHS} = \begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix}$$

On applying $R_2 \to R_2 - 2R_1, R_3 \to R_3 - 3R_1$, we get

$$\text{LHS} = \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & p-1 \\ 0 & 3 & 3p-2 \end{vmatrix}$$ **(1½)**

Now, on expanding along C_1, we get

$$\text{LHS} = 1 \times \begin{vmatrix} 1 & p-1 \\ 3 & 3p-2 \end{vmatrix}$$

$= 1[(3p-2) - (3p-3)]$ **(1½)**

$= 3p - 2 - 3p + 3 = 1$ **(1)**

$= \text{RHS}$ **Hence proved.**

36. Using properties of determinants, prove that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3.$$ **Delhi 2009, 2008**

To prove,

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

$$\text{LHS} = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

On applying $C_1 \to C_1 - bC_3$ and $C_2 \to C_2 + aC_3$, we get

$$\text{LHS} = \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b(1+a^2+b^2) & -a(1+a^2+b^2) & 1-a^2-b^2 \end{vmatrix}$$ **(1½)**

On taking $(1+a^2+b^2)$ common from C_1 and C_2, we get

$$\text{LHS} = (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2-b^2 \end{vmatrix}$$ **(1)**

On expanding along R_1, we get

$$\text{LHS} = (1+a^2+b^2)^2 \times [1(1-a^2-b^2+2a^2) - 2b(0-b)]$$

$$= (1+a^2+b^2)^2 (1+a^2+b^2)$$

$$= (1+a^2+b^2)^3 = \text{RHS}$$ **(1½)**

Hence proved.

37. Prove that $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3-3abc.$ Delhi 2009

To prove, $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$

$= a^3 + b^3 + c^3 - 3abc$

$$\text{LHS} = \begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$$

On applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\text{LHS} = \begin{vmatrix} a+b+c & b & c \\ 0 & b-c & c-a \\ 2(a+b+c) & c+a & a+b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 2 & c+a & a+b \end{vmatrix} \quad \textbf{(1)}$$

[$\because$ taking $(a+b+c)$ common from C_1]

On applying $R_3 \to R_3 - 2R_1$, we get

LHS = (a + b + c)
$$\begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 0 & c+a-2b & a+b-2c \end{vmatrix}$$

On expanding along C_1, we get

$$\text{LHS} = (a+b+c)\cdot 1\begin{vmatrix} b-c & c-a \\ c+a-2b & a+b-2c \end{vmatrix}$$

(1½)

On applying $R_2 \to R_2 + 2R_1$, we get

$$\text{LHS} = (a+b+c)\begin{vmatrix} b-c & c-a \\ a-c & b-a \end{vmatrix}$$

$= (a+b+c)\,[(b-c)(b-a) - (a-c)(c-a)]$

$= (a+b+c)\,[(b^2 - ab - bc + ac)$

$+ (a^2 + c^2 - 2ca)]$

$= (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= a^3 + b^3 + c^3 - 3abc = \text{RHS}$ **(1½)**

Hence proved.

38. Show that, if $x \neq y \neq z$ and

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0, \text{ then } 1 + xyz = 0.$$

Delhi 2008C; HOTS

Firstly, apply properties of determinants in LHS and reduce it into simplest form. Then, equate the lowest term to zero and use the given fact that $x \neq y \neq z$ to get the desired result.

Given, $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$ and $x \neq y \neq z$

The given determinant can be written as

$$\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0 \quad \textbf{(1/2)}$$

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

[taking x, y and z common from R_1, R_2 and R_3, respectively in second determinant] **(1/2)**

On applying $C_2 \leftrightarrow C_3$ in first determinant, we get

$$-\begin{vmatrix} x & 1 & x^2 \\ y & 1 & y^2 \\ z & 1 & z^2 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0 \quad \textbf{(1/2)}$$

On applying $C_1 \leftrightarrow C_2$ in first determinant, we get

$$(-)(-)\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} (1+xyz) = 0$$

Now, on applying $R_1 \rightarrow R_1 - R_2$ and $R_2 \rightarrow R_2 - R_3$, we get

$$\begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix}(1+xyz)=0$$

$$\Rightarrow (1+xyz)\begin{vmatrix} 0 & x-y & (x-y)(x+y) \\ 0 & y-z & (y-z)(y+z) \\ 1 & z & z^2 \end{vmatrix}=0$$

On taking $(x-y)$ common from R_1 and $(y-z)$ from R_2, we get

$$(1+xyz)(x-y)(y-z)\begin{vmatrix} 0 & 1 & x+y \\ 0 & 1 & y+z \\ 1 & z & z^2 \end{vmatrix}=0$$

(1½)

On expanding along C_1, we get

$(1+xyz)(x-y)(y-z)[1\times(y+z)-(x+y)]=0$

$\Rightarrow (1+xyz)(x-y)(y-z)(z-x)=0$

$\Rightarrow$ Either $1+xyz=0$

or $x-y=y-z=z-x=0$

$\Rightarrow x=y=z$

But this is contradiction as given that

$x\neq y\neq z$

$\therefore \quad 1+xyz=0$ **(1)**

Hence proved.

39. Using properties of determinants, prove that

$$\begin{vmatrix} 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix}=(x+y+z)^3.$$

$$\text{LHS} = \begin{vmatrix} 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix}$$

On applying $R_1' \to R_1 + R_2 + R_3$, we get

$$\text{LHS} = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix}$$

On taking $(x+y+z)$ common from R_1, we get

$$\text{LHS} = (x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix}$$

(1)

On applying $C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$, we get

$$\text{LHS} = (x+y+z)\begin{vmatrix} 1 & 0 & 0 \\ 2z & 0 & -(x+y+z) \\ x-y-z & x+y+z & x+y+z \end{vmatrix} \quad \textbf{(1)}$$

On taking $(x+y+z)$ common from C_2 and C_3, both, we get

$$\text{LHS} = (x+y+z)^3\begin{vmatrix} 1 & 0 & 0 \\ 2z & 0 & -1 \\ x-y-z & 1 & 1 \end{vmatrix} \quad \textbf{(1)}$$

$$= (x+y+z)^3\,[1(0+1)]$$

$$= (x+y+z)^3 = \text{RHS} \quad \textbf{(1)}$$

Hence proved.

Inverse of a Matrix and Application of Determinants and Matrix

Previous Years Examination Questions

6 Marks Questions

1. Two schools P and Q want to award their selected students on the values of discipline, politeness and punctuality. The school P wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to 3, 2 and 1 students respectively with a total award money of ₹ 1000. School Q wants to spend ₹ 1500 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as before). If the total amount of award for one prize on each value is ₹ 600, using matrices, find the award money for each value.
Apart from the above three values, suggest one more value for awards.

Value Based Question; Delhi 2014

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ 5 & -5 & -5 \end{bmatrix}^T = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

Then, $A^{-1} = \frac{1}{|A|}(\text{adj } A)$

$$= \frac{1}{-5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix}$$

(1)

Now, the solution of given system is given by

$$X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix}\begin{bmatrix} 1000 \\ 1500 \\ 600 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 2000 + 1500 - 3000 \\ 1000 - 3000 + 3000 \\ -3000 + 1500 + 3000 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 500 \\ 1000 \\ 1500 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 100 \\ 200 \\ 300 \end{bmatrix}$$ **(1)**

On comparing corresponding elements, we get $x = 100$, $y = 200$ and $z = 300$

Honesty is one more value which is also considered for the award. **(1)**

2. Two schools A and B want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school A wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values of 3, 2 and 1 students, respectively with a total award money of ₹ 1600. School B wants to spend ₹ 2300 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as before). If the total amount of award for one prize on each value is ₹ 900, using matrices, find the award money for each value. Apart from these three values, suggest one more value which should be considered for award. **All India 2014; Value Based Question**

Let the amount awarded to the students on th values of sincerity, truthfulness an helpfulness be ₹ x, ₹ y and ₹ z, respectively Then, according to the question,

$$3x + 2y + z = 1600$$
$$4x + y + 3z = 2300$$
and $$x + y + z = 900$$

This system of equations can be written a $AX = B$, where

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$$

Here, $|A| = \begin{vmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{vmatrix}$

$= 3(1-3) - 2(4-3) + 1(4-1)$
$= 3(-2) - 2(1) + 1(3) = -6 - 2 + 3$
$= -8 + 3 = -5 \neq 0$ **(1)**

Thus, A is non-singular matrix.

$\therefore A^{-1}$ exists.

Cofactors of $|A|$ are

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = (1)(-2) = -2$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 4 & 3 \\ 1 & 1 \end{vmatrix} = (-1)(1) = -1$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 4 & 1 \\ 1 & 1 \end{vmatrix} = (1)(3) = 3$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = (-1)(1) = -1$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} = (1)(2) = 2$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = (-1)(1) = -1$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = (1)(5) = 5$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 3 & 1 \\ 4 & 3 \end{vmatrix} = (-1)(5) = -5$$

and $A_{33} = (-1)^{3+3}\begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} = (1)(-5) = -5$ **(1)**

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ 5 & -5 & -5 \end{bmatrix}^T = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

Then, $A^{-1} = -\frac{1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$

$$\left[\because A^{-1} = \frac{1}{|A|}\text{adj}(A)\right]$$

$$= \frac{1}{5}\begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix}$$ **(1)**

Now, given system has a unique solution given by

$$X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix}\begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 3200 + 2300 - 4500 \\ 1600 - 4600 + 4500 \\ -4800 + 2300 + 4500 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 1000 \\ 1500 \\ 2000 \end{bmatrix} = \begin{bmatrix} 200 \\ 300 \\ 400 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 200 \\ 300 \\ 400 \end{bmatrix}$$

On comparing the corresponding elements, we get

$x = 200, y = 300, z = 400$ **(1)**

Hence, the amount of money for each value sincerity, truthfulness are helpfulness are ₹ 200, ₹ 300 and ₹ 400, respectively. **(1)**

Apart from these three values, punctuality should be considered for the award. **(1)**

3. Two schools P and Q want to award their selected students on the values of tolerance, kindness and leadership. The school P wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to 3, 2 and 1 students respectively with a total award money of ₹ 2200. School Q wants to spend ₹ 3100 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as school P). If the total amount of award for one prize on each value is ₹ 1200, using matrices, find the award money for each value.

Apart from these three values, suggest one more value which should be considered for award. **Foreign 2014; Value Based Question**

Let the amount awarded to the students on the values of Tolerance, kindness and leadership be ₹ x, ₹ y and ₹ z respectively, then according to the question,

$$3x + 2y + z = 2200$$
$$4x + y + 3z = 3100$$
$$x + y + z = 1200$$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$ **(1)**

Here, $|A| = \begin{vmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{vmatrix}$

$= 3(1-3) - 2(4-3) + 1(4-1)$ **(1)**

$= -6 - 2 + 3$

$= -5 \neq 0$

So, A is non-singular and its inverse exists.

Now, cofactors of $|A|$ are

$$A_{11} = (-1)^2 (1-3) = -2$$
$$A_{12} = (-1)^3 (4-3) = -1$$
$$A_{13} = (-1)^4 (4-1) = 3$$
$$A_{21} = (-1)^3 (2-1) = -1$$
$$A_{22} = (-1)^4 (3-1) = 2$$
$$A_{23} = (-1)^5 (3-2) = -1$$
$$A_{31} = (-1)^4 (6-1) = 5$$
$$A_{32} = (-1)^5 (9-4) = -5$$
$$A_{33} = (-1)^6 (3-8) = -5 \qquad \textbf{(1)}$$

$$\therefore \text{ adj } (A) = \begin{bmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ 5 & -5 & -5 \end{bmatrix}^T = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$\text{Then, } A^{-1} = \frac{1}{|A|} (\text{adj } A) = \frac{1}{-5} \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix} \qquad \textbf{(1)}$$

Now, the solution of given system is given by $X = A^{-1}B$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix} \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 1500 \\ 2000 \\ 2500 \end{bmatrix} = \begin{bmatrix} 300 \\ 400 \\ 500 \end{bmatrix} \qquad \textbf{(1)}$$

On comparing the corresponding elements, we get $x = 300$, $y = 400$ and $z = 600$

Apart from these three values, sincerity should be considered for the award. **(1)**

4. A total amount of ₹ 7000 is deposited in three different savings bank accounts with annual interest rates of 5%, 8% and $8\frac{1}{2}$% respectively. The total annual interest from these three accounts is ₹ 550. Equal amounts have been deposited in the 5% and 8% savings accounts. Find the amount deposited in each of the three accounts, with the help of matrices. **Delhi 2014C**

Let ₹ x, ₹y and ₹z be invested in saving accounts at the rate of 5%, 8% and $8\frac{1}{2}$%, respectively.

Then, the system of equations is

$$x + y + z = 7000 \quad \text{...(i)}$$

and $\frac{5x}{100} + \frac{8y}{100} + \frac{17z}{200} = 550$

$\Rightarrow \quad 10x + 16y + 17z = 110000 \quad$ ...(ii)

and $\quad x - y = 0 \quad$ ...(iii)

This system of equation can be written as $AX = B$

where, $A = \begin{bmatrix} 1 & 1 & 1 \\ 10 & 16 & 17 \\ 1 & -1 & 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

and $\quad B = \begin{bmatrix} 7000 \\ 110000 \\ 0 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 10 & 16 & 17 \\ 1 & -1 & 0 \end{vmatrix}$ **(1)**

$\Rightarrow \quad |A| = 1(0 + 17) - 1(0 - 17) + 1(-10 - 16)$

$= 17 + 17 - 26 = 8 \neq 0$ **(1)**

$\therefore A^{-1}$ exists.

Now, cofactors of $|A|$ are

$A_{11} = (-1)^2 \begin{vmatrix} 16 & 17 \\ -1 & 0 \end{vmatrix} = 1(0 + 17) = 17$

$$\begin{vmatrix} -1 & 0 \end{vmatrix}$$

$$A_{12} = (-1)^3 \begin{vmatrix} 10 & 17 \\ 1 & 0 \end{vmatrix} = -1(0-17) = 17$$

$$A_{13} = (-1)^4 \begin{vmatrix} 10 & 16 \\ 1 & -1 \end{vmatrix} = 1(-10-16) = -26$$

$$A_{21} = (-1)^3 \begin{vmatrix} 1 & 1 \\ -1 & 0 \end{vmatrix} = -1(0+1) = -1$$

$$A_{22} = (-1)^4 \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = 1(0-1) = -1$$

$$A_{23} = (-1)^5 \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1(-1-1) = 2$$

$$A_{31} = (-1)^4 \begin{vmatrix} 1 & 1 \\ 16 & 17 \end{vmatrix} = 1(17-16) = 1$$

$$A_{32} = (-1)^5 \begin{vmatrix} 1 & 1 \\ 10 & 17 \end{vmatrix} = -1(17-10) = -7$$

$$A_{33} = (-1)^6 \begin{vmatrix} 1 & 1 \\ 10 & 16 \end{vmatrix} = 1(16-10) = 6 \quad \textbf{(1)}$$

$$\therefore \ \text{adj}(A) = \begin{bmatrix} 17 & 17 & -26 \\ -1 & -1 & 2 \\ 1 & -7 & 6 \end{bmatrix}^T = \begin{bmatrix} 17 & -1 & 1 \\ 17 & -1 & -7 \\ -26 & 2 & 6 \end{bmatrix}$$

Then, $$A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{1}{8}\begin{bmatrix} 17 & -1 & 1 \\ 17 & -1 & -7 \\ -26 & 2 & 6 \end{bmatrix} \quad \textbf{(1)}$$

The solution of given system is given by

$X = A^{-1} \cdot B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 17 & -1 & 1 \\ 17 & -1 & -7 \\ -26 & 2 & 6 \end{bmatrix}\begin{bmatrix} 7000 \\ 110000 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 9000 \\ 9000 \\ 38000 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1125 \\ 1125 \\ 4750 \end{bmatrix} \qquad \textbf{(1)}$$

On comparing the corresponding elements, we get $x = 1125, y = 1125, z = 4750$.

Hence, the amount deposited in each type of account is ₹1125, ₹1125 and ₹4750, respectively. **(1)**

5. Two schools, P and Q, want to award their selected students for the values of sincerity, truthfulness and hard work at the rate of ₹ x, ₹ y and ₹ z for each respective value per student. School P awards its 2, 3 and 4 students on the above respective values with a total total prize money of ₹ 4600. School Q wants to award its 3, 2 and 3 students on the respective values with a total award money of ₹ 4100. If the total amount of award money for one prize on each value is ₹ 1500, using matrices find the award money for each value. Suggest one other value which the school can consider for awarding the students. **All India 2014C; Value Based Question**

Let the amount awarded to the students on the values of sincerity, truthfulness and hard work be ₹ x, ₹ y and ₹ z respectively, then according to question

$$2x + 3y + 4z = 4600$$
$$3x + 2y + 3z = 4100$$
$$x + y + z = 1500$$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 4600 \\ 4100 \\ 1500 \end{bmatrix}$$ **(1)**

Here, $|A| = \begin{vmatrix} 2 & 3 & 4 \\ 3 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix}$

$$= 2(2-3) - 3(3-3) + 4(3-2)$$
$$= -2 - 0 + 4 = 2 \neq 0$$ **(1)**

So, A is non-singular and its inverse exists.

Now, cofactors of $|A|$ are

$$A_{11} = (-1)^2 (2-3) = -1$$
$$A_{12} = (-1)^3 (3-3) = 0$$
$$A_{13} = (-1)^4 (3-2) = 1$$
$$A_{21} = (-1)^3 (3-4) = 1$$
$$A_{22} = (-1)^4 (2-4) = -2$$
$$C_{23} = (-1)^5 (2-3) = 1$$
$$A_{31} = (-1)^4 (9-8) = 1$$
$$A_{32} = (-1)^5 (6-12) = 6$$

$$A_{33} = (-1)^6\,(4-9) = -5 \qquad \textbf{(1)}$$

$$\therefore \text{adj}\,(A) = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -2 & 1 \\ 1 & 6 & -5 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 & 1 \\ 0 & -2 & 6 \\ 1 & 1 & -5 \end{bmatrix}$$

Then, $A^{-1} = \frac{1}{|A|}(\text{adj}\,A) = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 0 & -2 & 6 \\ 1 & 1 & -5 \end{bmatrix}$ **(1)**

Now, the solution of given system is given by

$$X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 0 & -2 & 6 \\ 1 & 1 & -5 \end{bmatrix}\begin{bmatrix} 4600 \\ 4100 \\ 1500 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1000 \\ 800 \\ 1200 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 500 \\ 400 \\ 600 \end{bmatrix}$$

On comparing the corresponding elements, we get $x = 500$, $y = 400$ and $z = 600$. **(1)**

Apart from these three values, punctuality should be considered for the award. **(1)**

6. Two institutions decided to award their employees for the three values of resourcefulness, competence and determination in the form of prizes at the rate of ₹ x, ₹ y and ₹ z respectively per person. The first institution decided to award respectively 4, 3 and 2 employees with a total prize money of ₹ 37000 and the second institution decided to award respectively 5, 3 and 4 employees with a total prize money of ₹ 47000. If all the three prizes per person together amount to ₹ 12000, then using matrix method, find the values of x, y and z. What values are described in this question?

Delhi 2013C; Value Based Question

Given, two institutions decided to award their employees for the three values of resourcefulness, competence and determination in the form of prizes at the rate of ₹ x, ₹ y and ₹ z, respectively.

Then, according to the given condition,

$$4x + 3y + 2z = 37000$$

$$5x + 3y + 4z = 47000$$

and $$x + y + z = 12000$$ **(1)**

The system of equations can be written in matrix form as

$$\begin{bmatrix} 4 & 3 & 2 \\ 5 & 3 & 4 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 37000 \\ 47000 \\ 12000 \end{bmatrix}$$

i.e. $AX = B$, where ...(i)

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 5 & 3 & 4 \\ 1 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 37000 \\ 47000 \\ 12000 \end{bmatrix} \text{ and } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$ **(1)**

Now, $|A| = 4(3 - 4) - 3(5 - 4) + 2(5 - 3)$

$= -4 - 3 + 4 = -3 \neq 0$

So, A is non-singular and its inverse exists.

Now, cofactors of $|A|$ are

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 3 & 4 \\ 1 & 1 \end{vmatrix} = +(3-4) = -1$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 5 & 4 \\ 1 & 1 \end{vmatrix} = -(5-4) = -1$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 5 & 3 \\ 1 & 1 \end{vmatrix} = +(5-3) = 2$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = -(3-2) = -1$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 4 & 2 \\ 1 & 1 \end{vmatrix} = +(4-2) = 2$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 4 & 3 \\ 1 & 1 \end{vmatrix} = -(4-3) = -1$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 3 & 2 \\ 3 & 4 \end{vmatrix} = +(12-6) = 6$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 4 & 2 \\ 5 & 4 \end{vmatrix} = -(16-10) = -6$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 4 & 3 \\ 5 & 3 \end{vmatrix} = +(12-15) = -3 \quad \textbf{(1)}$$

$$\therefore \text{adj}(A) = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} -1 & -1 & 6 \\ -1 & 2 & -6 \\ 2 & -1 & -3 \end{bmatrix}$$

Then, $$A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{1}{-3}\begin{bmatrix} -1 & -1 & 6 \\ -1 & 2 & -6 \\ 2 & -1 & -3 \end{bmatrix} \quad \textbf{(1)}$$

Now, solution of Eq. (i) is given by $X = A^{-1}B$

$$\therefore \quad X = -\frac{1}{3}\begin{bmatrix} -1 & -1 & 6 \\ -1 & 2 & -6 \\ 2 & -1 & -3 \end{bmatrix}\begin{bmatrix} 37000 \\ 47000 \\ 12000 \end{bmatrix}$$

$$= -\frac{1}{3}\begin{bmatrix} -37000 - 47000 + 72000 \\ -37000 + 94000 - 72000 \\ 74000 - 47000 - 36000 \end{bmatrix}$$

$$= -\frac{1}{3}\begin{bmatrix} -12000 \\ -15000 \\ -9000 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4000 \\ 5000 \\ 3000 \end{bmatrix}$$

On comparing corresponding elements, we get

$x = 4000, y = 5000$ and $z = 3000$ **(1)**

The value described in the question are resourcefulness, competence and determination. **(1)**

7. A school wants to award its students for the values of honesty, regularity and hard work with a total cash award of ₹ 6000. Three times the award money for hard work added to that given for honesty, amounts to ₹ 11000. The award money given for honesty and hard work together is double the one given for regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values, namely, honesty, regularity and hard work, suggest one more value which the school must include for award.

Value Based Question; Delhi 2013

> Consider x, y and z are the award of honesty, regularity and hard work and form the system of equations. Then, write them in matrix form as $AX = B$. Now, the solution is given by $X = A^{-1}B$, put the values of A^{-1}, X and B and calculate the required values.

Let award for honesty $= ₹\ x$

Award for regularity $= ₹\ y$

and award for hard work $= ₹\ z$

and award for hard work = ₹ z

According to first condition,

$$x+y+z=6000$$

According to second condition,

$$3z+x=11000$$

According to third condition,

$$x+z=2y$$

Now, the above equations can be rewritten in standard form of linear equations as

$$x+y+z=6000, \quad \text{...(i)}$$

$$x+0y+3z=11000 \quad \text{...(ii)}$$

and $$x-2y+z=0 \quad \text{...(iii)} \; \textbf{(1)}$$

We can represent these equations using matrices as

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$$

i.e. $AX = B$,

where, $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$

and its solution is given by

$$X = A^{-1}B \quad \text{...(iv)}$$

Now, $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{vmatrix}$

$$= 1(0+6) - 1(1-3) + 1(-2-0)$$

$$= 6+2-2 = 6 \neq 0 \quad \textbf{(1)}$$

$\therefore A^{-1}$ exists, because A is a non-singular matrix.

Now, cofactors of $|A|$ are

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 0 & 3 \\ -2 & 1 \end{vmatrix} = +(0+6) = +6$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = -(1-3) = 2$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 1 & 0 \\ 1 & -2 \end{vmatrix} = +(-2) = -2$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix} = -(1+2) = -3$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = +(1-1) = 0$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = -(-2-1) = 3$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} = +(3-0) = 3$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = -(3-1) = -2$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = +(0-1) = -1$$

Now, $C = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 6 & 2 & -2 \\ -3 & 0 & 3 \\ 3 & -2 & -1 \end{bmatrix}$

(1)

$\therefore \text{adj}(A) = C^T = \begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$

Then, $A^{-1} = \frac{1}{|A|}(\text{adj}) = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$ **(1)**

Now, $X = A^{-1}B = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}\begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$

$= \frac{1}{6}\begin{bmatrix} 36000 - 33000 + 0 \\ 12000 + 0 - 0 \end{bmatrix}$

$$6\begin{bmatrix}-12000+33000-0\end{bmatrix}$$

$$=\frac{1}{6}\begin{bmatrix}3000\\12000\\21000\end{bmatrix}=\begin{bmatrix}500\\2000\\3500\end{bmatrix}$$

$$\Rightarrow \qquad X=\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}500\\2000\\3500\end{bmatrix}$$

On comparing, we get

$$x = 500,\ y = 2000 \text{ and } z = 3500$$

Hence, award for honesty = ₹ 500

Award for regularity = ₹ 2000

and award for hard work = ₹ 3500 **(1½)**

The school must include punctuality for award. **(1/2)**

8. The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some others (say z) for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method, find the number of awardees of each category. Apart from these values, namely, honesty, cooperation and supervision, suggest one more value which the management of the colony must include for awards.

Value Based Question; All India 2013

Given,

Number of members for honesty = x

Number of members for helping others = y

and number of members for supervising the workers to keep the colony neat and clean = z

Now, by first condition,

$$x + y + z = 12 \quad \text{...(i)}$$

By second condition,

$$2x + 3(y + z) = 33$$

$$\Rightarrow \quad 2x + 3y + 3z = 33 \quad \text{...(ii)}$$

and by third condition,

$$(x + z) = 2y$$

$$\Rightarrow \quad x - 2y + z = 0 \quad \text{...(iii)}$$

$\therefore$ System of equations becomes

$$x + y + z = 12$$

$$2x + 3y + 3z = 33$$

and $$x - 2y + z = 0$$ **(1)**

In matrix form, it can be written as

$$AX = B \quad \Rightarrow \quad X = A^{-1}B \quad \text{...(iv)}$$

where, matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}$

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$

Now, $|A| = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}$

$$= 1(3 + 6) - 1(2 - 3) + 1(-4 - 3)$$

$$= 9 + 1 - 7 = 10 - 7 = 3 \neq 0$$ **(1)**

So, A^{-1} exists.

Cofactors of $|A|$ are

$$A_{11}=(-1)^{1+1}\begin{vmatrix}3 & 3\\ -2 & 1\end{vmatrix}=3+6=9$$

$$A_{12}=(-1)^{1+2}\begin{vmatrix}2 & 3\\ 1 & 1\end{vmatrix}=2-3=1$$

$$A_{13}=(-1)^{1+3}\begin{vmatrix}2 & 3\\ 1 & -2\end{vmatrix}=-4-3=-7$$

$$A_{21}=(-1)^{2+1}\begin{vmatrix}1 & 1\\ -2 & 1\end{vmatrix}=1+2=-3$$

$$A_{22}=(-1)^{2+2}\begin{vmatrix}1 & 1\\ 1 & 1\end{vmatrix}=1-1=0$$

$$A_{23}=(-1)^{2+3}\begin{vmatrix}1 & 1\\ 1 & -2\end{vmatrix}=-2-1=3$$

$$A_{31}=(-1)^{3+1}\begin{vmatrix}1 & 1\\ 3 & 3\end{vmatrix}=3-3=0$$

$$A_{32}=(-1)^{3+2}\begin{vmatrix}1 & 1\\ 2 & 3\end{vmatrix}=3-2=-1$$

$$A_{33}=(-1)^{3+3}\begin{vmatrix}1 & 1\\ 2 & 3\end{vmatrix}=3-2=1$$

$$\therefore \quad C=\begin{bmatrix}A_{11} & A_{12} & A_{13}\\ A_{21} & A_{22} & A_{23}\\ A_{31} & A_{32} & A_{33}\end{bmatrix}=\begin{bmatrix}9 & 1 & -7\\ -3 & 0 & 3\\ 0 & -1 & 1\end{bmatrix} \quad \textbf{(1)}$$

Now, $\text{adj}(A)=C^T=\begin{bmatrix}9 & -3 & 0\\ 1 & 0 & -1\\ -7 & 3 & 1\end{bmatrix}$

Then, $A^{-1}=\dfrac{\text{adj}(A)}{|A|}=\dfrac{1}{3}\begin{bmatrix}9 & -3 & 0\\ 1 & 0 & -1\\ -7 & 3 & 1\end{bmatrix}$ **(1/2)**

Now, from Eq. (iv), we have

$$X=A^{-1}B$$

$$X=\frac{1}{3}\begin{bmatrix}9 & -3 & 0\\ 1 & 0 & -1\\ \ldots\end{bmatrix}\begin{bmatrix}12\\ 33\\ \ldots\end{bmatrix}$$

$$\begin{bmatrix} -7 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 108-99 \\ 12 \\ -84+99 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9 \\ 12 \\ 15 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$$\Rightarrow \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

On comparing corresponding elements, we get

$$x = 3, \quad y = 4 \quad \text{and} \quad z = 5$$ **(1½)**

The management of the colony must include the bravery award for some of its members. Because in this category, we appreciate the brave members of the colony for their bravery and make aware the other members (men, women and children) of the colony. **(1)**

9. Using matrices, solve the following system of equations.

$$x - y + 2z = 7$$
$$3x + 4y - 5z = -5$$
and $$2x - y + 3z = 12$$ **Delhi 2012**

> Given, system of equations can be written in matrix form $AX = B$. So firstly, determine the cofactors of A and then determine A^{-1} and then use the relation $X = A^{-1}B$, to get the values of x, y and z.

Given, system of is equations are

$$x - y + 2z = 7$$
$$3x + 4y - 5z = -5$$
and
$$2x - y + 3z = 12$$

In matrix form, it can be written as

$X = AB \Rightarrow X = A^{-1}B$ ...(i) **(1)**

where,

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

Now, $|A| = 1(12 - 5) + 1(9 + 10) + 2(-3 - 8)$

$= 1(7) + 1(19) + 2(-11)$ **(1)**

$= 7 + 19 - 22 = 4$

$\Rightarrow$ $|A| \neq 0$, hence A^{-1} exists.

Now, cofactors of $|A|$ are

$$A_{11} = (-1)^2 \begin{vmatrix} 4 & -5 \\ -1 & 3 \end{vmatrix} = 1(12 - 5) = 7$$

$$A_{12} = (-1)^3 \begin{vmatrix} 3 & -5 \\ 2 & 3 \end{vmatrix} = -1(9 + 10) = -19$$

$$A_{13} = (-1)^4 \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = 1(-3 - 8) = -11$$

$$A_{21} = (-1)^3 \begin{vmatrix} -1 & 2 \\ -1 & 3 \end{vmatrix} = -1(-3 + 2) = 1$$

$$A_{22} = (-1)^4 \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 1(3 - 4) = -1$$

$$A_{23} = (-1)^5 \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = -1(-1 + 2) = -1$$

$$A = (-1)^4 \begin{vmatrix} -1 & 2 \end{vmatrix} = 1(5 - 8) = -3$$

$$A_{31} = (-1)^{4}\begin{vmatrix} & \\ 4 & -5 \end{vmatrix} = 1(5-8) = -3$$

$$A_{32} = (-1)^5\begin{vmatrix} 1 & 2 \\ 3 & -5 \end{vmatrix} = -1(-5-6) = 11$$

$$A_{33} = (-1)^6\begin{vmatrix} 1 & -1 \\ 3 & 4 \end{vmatrix} = 1(4+3) = 7$$

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 7 & -19 & -11 \\ 1 & -1 & -1 \\ -3 & 11 & 7 \end{bmatrix}^T$$

$$= \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix} \qquad \textbf{(1½)}$$

Then, $A^{-1} = \dfrac{\text{adj}(A)}{|A|}$

$$\Rightarrow \quad A^{-1} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix} \qquad \textbf{(1/2)}$$

On putting the value of X, A^{-1} and B in Eq. (i), we get

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix} \qquad \textbf{(1)}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 49-5-36 \\ -133+5+132 \\ -77+5+84 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

On comparing corresponding elements, we get

$$x = 2, \; y = 1 \text{ and } z = 3 \qquad \textbf{(1)}$$

10. Using matrices, solve the following system of linear equations.

$$x + y - z = 3$$
$$2x + 3y + z = 10$$
and $$3x - y - 7z = 1$$ **All India 2012**

Given system of equations is

$$x + y - z = 3;\ 2x + 3y + z = 10$$

and $3x - y - 7z = 1$

In matrix form, it can be written as

$$AX = B \Rightarrow X = A^{-1}B \qquad \text{...(i)} \ \mathbf{(1)}$$

where,

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$$

Now, $|A| = 1(-21+1) - 1(-14-3) \cdot 1(-2-9)$

$= 1(-20) - 1(-17) - 1(-11)$ **(1)**

$= -20 + 17 + 11 = 8$

$\Rightarrow$ $|A| \neq 0$, hence unique solution exists.

Cofactors of $|A|$ are

$$A_{11} = (-1)^2 \begin{vmatrix} 3 & 1 \\ -1 & -7 \end{vmatrix} = 1(-21+1) = -20$$

$$A_{12} = (-1)^3 \begin{vmatrix} 2 & 1 \\ 3 & -7 \end{vmatrix} = -1(-14-3) = 17$$

$$A_{13} = (-1)^4 \begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix} = 1(-2-9) = -11$$

$$A_{21} = (-1)^3 \begin{vmatrix} 1 & -1 \\ -1 & -7 \end{vmatrix} = -1(-7-1) = 8$$

$$A_{22} = (-1)^4 \begin{vmatrix} 1 & -1 \\ 3 & -7 \end{vmatrix} = 1(-7+3) = -4$$

$$A_{23} = (-1)^5 \begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} = -1(-1-3) = 4$$

$$A_{31} = (-1)^4 \begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix} = 1(1+3) = 4$$

$$A \ldots = (-1)^5 \begin{vmatrix} 1 & -1 \end{vmatrix} = -1(1+2) = -3$$

$$A_{32} = (-1)^5 \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1(1+2) = -3$$

$$A_{33} = (-1)^6 \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1(3-2) = 1 \quad \textbf{(1½)}$$

$$\therefore \text{adj}(A) = \begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix}^T$$

$$= \begin{bmatrix} -20 & 17 & -11 \\ 8 & -4 & 4 \\ 4 & -3 & 1 \end{bmatrix}^T = \begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix} \textbf{(1)}$$

Then, $A^{-1} = \dfrac{\text{adj}(A)}{|A|}$

$$\Rightarrow \quad A^{-1} = \frac{1}{8}\begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix} \quad \textbf{(1/2)}$$

Now, from Eq. (i), we have

$$X = \frac{1}{8}\begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -60+80+4 \\ 51-40-3 \\ -33+40+1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ 8 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

On comparing corresponding elements, we get

$$x = 3,\ y = 1 \text{ and } z = 1 \quad \textbf{(1)}$$

11. If $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}$, then find A^{-1} and hence solve the system of equations

$$x + 2y + z = 4$$

$$-x + y + z = 0$$

and $\quad x - 3y + z = 4.$ **Delhi 2012C**

Given, $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}$

$\because \quad |A| = 1(1+3) - 2(-1-1) + 1(3-1)$
$\quad = 4 + 4 + 2 = 10$

$\Rightarrow \quad |A| \neq 0$, hence unique solution exists. **(1)**

Now, cofactors of $|A|$ are

$$A_{11} = (-1)^2 \begin{vmatrix} 1 & 1 \\ -3 & 1 \end{vmatrix} = 1 + 3 = 4$$

$$A_{12} = (-1)^3 \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -(-1-1) = 2$$

$$A_{13} = (-1)^4 \begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} = 3 - 1 = 2$$

$$A_{21} = (-1)^3 \begin{vmatrix} 2 & 1 \\ -3 & 1 \end{vmatrix} = -(2+3) = -5$$

$$A_{22} = (-1)^4 \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1 - 1 = 0$$

$$A_{23} = (-1)^5 \begin{vmatrix} 1 & 2 \\ 1 & -3 \end{vmatrix} = -1(-3-2) = 5$$

$$A_{31} = (-1)^4 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 2 - 1 = 1$$

$$A_{32} = (-1)^5 \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = -(1+1) = -2$$

$$A_{33} = (-1)^6 \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} = 1 + 2 = 3$$

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}^T = \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$$ **(1½)**

Now, $A^{-1} = \frac{1}{|A|} \text{adj}(A)$

$$\therefore \quad A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$$ **(1/2)**

Given system of equations, can be written as

$$AX = B$$

where,

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix} \text{ and } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$ **(1)**

$$\therefore X = A^{-1}B = \frac{1}{10}\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16+0+4 \\ 8+0+(-8) \\ 8+0+12 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 20 \\ 0 \\ 20 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

(1)

On comparing corresponding elements, we get

$x = 2, y = 0$ and $z = 2$ **(1)**

12. Determine the product of $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$

$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and then use to solve the system of equations

$$x - y + z = 4$$
$$x - 2y - 2z = 9$$

and $\quad 2x + y + 3z = 1$. **Delhi 2012C; HOTS**

> Firstly, find the product of given matrices and then pre-multiply both sides of the product by A^{-1} and obtain A^{-1}. Then, by using A^{-1} and concept of matrix method, find the values of x, y and z.

Let $B = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$

$$\therefore BA = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$=\begin{bmatrix}-4+4+8 & 4-8+4 & -4-8+12\\ -7+1+6 & 7-2+3 & -7-2+9\\ 5-3-2 & -5+6-1 & 5+6-3\end{bmatrix}$$

$$=\begin{bmatrix}8 & 0 & 0\\ 0 & 8 & 0\\ 0 & 0 & 8\end{bmatrix}=8\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=8I \quad \textbf{(1½)}$$

$\Rightarrow$ $BA=8I \Rightarrow B(AA^{-1})=8IA^{-1}$

[multiplying both sides by A^{-1}]

$\Rightarrow$ $B=8A^{-1}$ $\quad [\because AA^{-1}=I]$

$$\therefore \quad A^{-1}=\frac{1}{8}\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix} \quad \textbf{(1)}$$

Given system of equations can be written in matrix form as

$$AX=C \Rightarrow X=A^{-1}C \quad \textbf{(1)}$$

where,

$$A=\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}, C=\begin{bmatrix}4\\ 9\\ 1\end{bmatrix} \text{ and } X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$$

(1/2)

$$\therefore \quad X=\frac{1}{8}\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}4\\ 9\\ 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{8}\begin{bmatrix}-16+36+4\\ -28+9+3\\ 20-27-1\end{bmatrix}=\frac{1}{8}\begin{bmatrix}24\\ -16\\ -8\end{bmatrix}=\begin{bmatrix}3\\ -2\\ -1\end{bmatrix}$$

(1)

On comparing corresponding elements, we get

$$x=3, y=-2 \text{ and } z=-1 \quad \textbf{(1)}$$

13. Find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$. Hence, solve the system of equations,

$$x + 2y - 3z = -4$$
$$2x + 3y + 2z = 2$$
and $$3x - 3y - 4z = 11.$$

All India 2012C, 2010, 2008

Given, $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$

$\therefore \quad |A| = 1(-12+6) - 2(-8-6) - 3(-6-9)$
$= -6 + 28 + 45 = 67$ **(1)**

$\Rightarrow \quad |A| \neq 0$, hence unique solution exists. **(1/2)**

Now, cofactors of $|A|$ are

$$A_{11} = (-1)^2 \begin{vmatrix} 3 & 2 \\ -3 & -4 \end{vmatrix} = -12 + 6 = -6$$

$$A_{12} = (-1)^3 \begin{vmatrix} 2 & 2 \\ 3 & -4 \end{vmatrix} = -(1)(-8-6) = 14$$

$$A_{13} = (-1)^4 \begin{vmatrix} 2 & 3 \\ 3 & -3 \end{vmatrix} = -6 - 9 = -15$$

$$A_{21} = (-1)^3 \begin{vmatrix} 2 & -3 \\ -3 & -4 \end{vmatrix} = -(-8-9) = 17$$

$$A_{22} = (-1)^4 \begin{vmatrix} 1 & -3 \\ 3 & -4 \end{vmatrix} = (-4+9) = 5$$

$$A_{23} = (-1)^5 \begin{vmatrix} 1 & 2 \\ 3 & -3 \end{vmatrix} = -(-3-6) = 9$$

$$A_{31} = (-1)^4 \begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix} = (4+9) = 13$$

$$A_{32} = (-1)^5 \begin{vmatrix} 1 & -3 \\ 2 & 2 \end{vmatrix} = -(2+6) = -8$$

$$A_{33} = (-1)^6 \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = (3 - 4) = -1$$

$$\therefore \text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} -6 & 14 & -15 \\ 17 & 5 & 9 \\ 13 & -8 & -1 \end{bmatrix}^T \quad \textbf{(1)}$$

$$= \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

Now, $A^{-1} = \frac{1}{|A|} \cdot \text{adj}(A)$

$$\therefore \quad A^{-1} = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} \quad \textbf{(1/2)}$$

Given system of equations can be written in matrix form as

$$AX = B \quad \Rightarrow X = A^{-1} B$$

where,

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}, \; B = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix} \text{and } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \textbf{(1)}$$

$$\therefore \quad X = A^{-1}B = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$= \frac{1}{67}\begin{bmatrix} 24+34+143 \\ -56+10-88 \\ +60+18-11 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67}\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} \qquad \textbf{(1)}$$

On comparing corresponding elements, we get

$$x = 3, y = -2 \text{ and } z = 1 \qquad \textbf{(1)}$$

14. Using matrix method, solve the following system of equations.

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

and $$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2, x, y, z \neq 0$$

Delhi 2011

Since, $|A| \neq 0$, so unique solution exists.

Now, cofactors of $|A|$ are

$$A_{11} = (-1)^2\begin{vmatrix} -6 & 5 \\ 9 & -20 \end{vmatrix} = 1(120 - 45) = 75$$

$$A_{12} = (-1)^3\begin{vmatrix} 4 & 5 \\ 6 & -20 \end{vmatrix} = -1(-80 - 30) = 110$$

$$A_{13} = (-1)^4\begin{vmatrix} 4 & -6 \\ 6 & 9 \end{vmatrix} = 1(36 + 36) = 72$$

$$A_{21} = (-1)^3\begin{vmatrix} 3 & 10 \\ 9 & -20 \end{vmatrix} = -1(-60 - 90) = 150$$

$$A_{22} = (-1)^4\begin{vmatrix} 2 & 10 \\ 6 & -20 \end{vmatrix} = 1(-40 - 60) = -100$$

$$A_{23} = (-1)^5 \begin{vmatrix} 2 & 3 \\ 6 & 9 \end{vmatrix} = -1(18-18) = 0$$

$$A_{31} = (-1)^4 \begin{vmatrix} 3 & 10 \\ -6 & 5 \end{vmatrix} = 1(15+60) = 75$$

$$A_{32} = (-1)^5 \begin{vmatrix} 2 & 10 \\ 4 & 5 \end{vmatrix} = -1(10-40) = 30$$

$$A_{33} = (-1)^6 \begin{vmatrix} 2 & 3 \\ 4 & -6 \end{vmatrix} = 1(-12-12) = -24$$

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}^T$$

$$= \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$ **(1)**

Then, $A^{-1} = \dfrac{\text{adj}(A)}{|A|}$

$$= \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$ **(1/2)**

On putting the values X, A^{-1} and B in Eq. (ii), we get

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 300+150+150 \\ 440-100+60 \\ 288+0-48 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$$

On comparing corresponding elements, we get

$$u = \frac{600}{1200}, \ v = \frac{400}{1200}, w = \frac{240}{1200} \quad \textbf{(1)}$$

$$\therefore \quad u = \frac{1}{2}, v = \frac{1}{3} \text{ and } w = \frac{1}{5}$$

But $\frac{1}{x} = u, \frac{1}{y} = v \text{ and } \frac{1}{z} = w$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{2}, \frac{1}{y} = \frac{1}{3} \text{ and } \frac{1}{z} = \frac{1}{5}$$

$$\therefore \quad x = 2, y = 3 \text{ and } z = 5 \quad \textbf{(1)}$$

15. Using matrices, solve the following system of equations.

$$4x + 3y + 2z = 60$$

$$x + 2y + 3z = 45$$

and $$6x + 2y + 3z = 70$$ **All India 2011**

The given system of equations can be written in matrix form as

$$AX = B$$

where, $A = \begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Its solution is given by

$$X = A^{-1}B \quad \ldots\text{(i)}$$

where, $A^{-1} = \dfrac{\text{adj}(A)}{|A|}$ **(1)**

Now, $|A| = 4 \times \begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix} - 3 \times \begin{vmatrix} 1 & 3 \\ 6 & 3 \end{vmatrix} + 2 \times \begin{vmatrix} 1 & 2 \\ 6 & 2 \end{vmatrix}$

$= 4(6-6) - 3(3-18) + 2(2-12)$ **(1)**

$= 4(0) - 3(-15) + 2(-10) = 0 + 45 - 20$

$= 25$

$\Rightarrow \quad |A| \neq 0$, hence unique solution exists. **(1)**

Now, cofactors of $|A|$ are

$A_{11} = (-1)^2 \begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix} = 1(6-6) = 0$

$A_{12} = (-1)^3 \begin{vmatrix} 1 & 3 \\ 6 & 3 \end{vmatrix} = -1(3-18) = -1(-15) = 15$

$A_{13} = (-1)^4 \begin{vmatrix} 1 & 2 \\ 6 & 2 \end{vmatrix} = 1(2-12) = 1(-10) = -10$

$A_{21} = (-1)^3 \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = -1(9-4) = -1(5) = -5$

$A_{22} = (-1)^4 \begin{vmatrix} 4 & 2 \\ 6 & 3 \end{vmatrix} = 1(12-12) = 0$

$A_{23} = (-1)^5 \begin{vmatrix} 4 & 3 \\ 6 & 2 \end{vmatrix} = -1(8-18) = -1(-10) = 10$

$A_{31} = (-1)^4 \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = 1(9-4) = 1(5) = 5$

$A_{32} = (-1)^5 \begin{vmatrix} 4 & 2 \\ 1 & 3 \end{vmatrix} = -1(12-2) = -1(10) = -10$

$$A_{33} = (-1)^6 \begin{vmatrix} 4 & 3 \\ 1 & 2 \end{vmatrix} = 1(8-3) = 5$$

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 15 & -10 \\ -5 & 0 & 10 \\ 5 & -10 & 5 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & -5 & 5 \\ 15 & 0 & -10 \\ -10 & 10 & 5 \end{bmatrix}$$ **(1½)**

Then, $$A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{1}{25}\begin{bmatrix} 0 & -5 & 5 \\ 15 & 0 & -10 \\ -10 & 10 & 5 \end{bmatrix}$$

(1/2)

From Eq. (i), we have

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{25}\begin{bmatrix} 0 & -5 & 5 \\ 15 & 0 & -10 \\ -10 & 10 & 5 \end{bmatrix}\begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$$

$$= \frac{1}{25}\begin{bmatrix} 0 - 225 + 350 \\ 900 + 0 - 700 \\ -600 + 450 + 350 \end{bmatrix} = \frac{1}{25}\begin{bmatrix} 125 \\ 200 \\ 200 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

On comparing corresponding elements, we get

$x = 5, y = 8$ and $z = 8$ **(1)**

16. Using matrices, solve the following system of equations.

$$x + 2y + z = 7$$
$$x + 3z = 11$$
and
$$2x - 3y = 1$$

All India 2011; Delhi 2008C

The given system of equations can be written in matrix form as $AX = B$

where,

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}, B = \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix} \text{ and } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Its solution is given by

$$X = A^{-1}B \quad \text{...(i)} \quad \textbf{(1)}$$

Now,

$$|A| = 1 \times \begin{vmatrix} 0 & 3 \\ -3 & 0 \end{vmatrix} - 2 \times \begin{vmatrix} 1 & 3 \\ 2 & 0 \end{vmatrix} + 1 \times \begin{vmatrix} 1 & 0 \\ 2 & -3 \end{vmatrix}$$
$$= 1(0 + 9) - 2(0 - 6) + 1(-3 - 0)$$
$$= 9 + 12 - 3 = 18$$

$\Rightarrow \quad |A| \neq 0$, hence unique solution exists. **(1)**

Now, cofactors of $|A|$ are

$$A_{11} = (-1)^2 \begin{vmatrix} 0 & 3 \\ -3 & 0 \end{vmatrix} = 1(0 + 9) = 9$$

$$A_{12} = (-1)^3 \begin{vmatrix} 1 & 3 \\ 2 & 0 \end{vmatrix} = -1(0 - 6) = -1(-6) = 6$$

$$A_{13} = (-1)^4 \begin{vmatrix} 1 & 0 \\ 2 & -3 \end{vmatrix} = 1(-3 - 0) = -3$$

$$A_{21} = (-1)^3 \begin{vmatrix} 2 & 1 \\ -3 & 0 \end{vmatrix} = -1(0 + 3) = -1(3) = -3$$

$$A_{22} = (-1)^4 \begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} = 1(0 - 2) = 1(-2) = -2$$

$$A_{23} = (-1)^5 \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix}$$

$= -1(-3-4) = -1(-7) = 7$

$$A_{31} = (-1)^4 \begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix} = 1(6-0) = 1(6) = 6$$

$$A_{32} = (-1)^5 \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = -1(3-1) = -1(2) = -2$$

$$A_{33} = (-1)^6 \begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} = 1(0-2) = 1(-2) = -2$$

$$\therefore \text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 9 & 6 & -3 \\ -3 & -2 & 7 \\ 6 & -2 & -2 \end{bmatrix}^T = \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$$ **(1½)**

$$\therefore \quad A^{-1} = \frac{1}{|A|} \cdot \text{adj}(A) = \frac{1}{18} \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$$

(1/2)

From Eq. (i), we have

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{18} \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix} \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$$ **(1)**

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{18} \begin{bmatrix} 63-33+6 \\ 42-22-2 \\ -21+77-2 \end{bmatrix} = \frac{1}{18} \begin{bmatrix} 36 \\ 18 \\ 54 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

On comparing corresponding elements,
$x = 2, y = 1$ and $z = 3$. **(1)**

17. Using matrices, solve the following system of equations.

$$x + 2y - 3z = -4$$
$$2x + 3y + 2z = 2$$
and $$3x - 3y - 4z = 11.$$ **All India 2011, 2008**

The given system of equations can be written in matrix form as $AX = B$,

where $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, $B = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Its solution is given by, $X = A^{-1}B$ ...(i) **(1)**

Now, $|A| = 1(-12 + 6) - 2(-8 - 6) - 3(-6 - 9)$

$= 1(-6) - 2(-14) - 3(-15)$

$= -6 + 28 + 45 = 67$

$\Rightarrow$ $|A| \neq 0$, hence unique solution exists. **(1)**

Now, cofactors of $|A|$ are

$$A_{11} = (-1)^2 \begin{vmatrix} 3 & 2 \\ -3 & -4 \end{vmatrix} = 1(-12 + 6) = -6$$

$$A_{12} = (-1)^3 \begin{vmatrix} 2 & 2 \\ 3 & -4 \end{vmatrix} = -1(-8 - 6)$$

$$= -1(-14) = 14$$

$$A_{13} = (-1)^4 \begin{vmatrix} 2 & 3 \\ 3 & -3 \end{vmatrix} = 1(-6 - 9)$$

$$= 1(-15) = -15$$

$$A_{21} = (-1)^3 \begin{vmatrix} 2 & -3 \\ -3 & -4 \end{vmatrix} = -1(-8 - 9)$$

$$= -1(-17) = 17$$

$$A_{22} = (-1)^4 \begin{vmatrix} 1 & -3 \\ 3 & -4 \end{vmatrix} = 1(-4 + 9) = 1(5) = 5$$

$$A_{23} = (-1)^5 \begin{vmatrix} 1 & 2 \\ 3 & -3 \end{vmatrix} = -1(-3 - 6)$$

$$= -1(-9) = 9$$

$$A_{31} = (-1)^4 \begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix} = 1(4+9) = 13$$

$$A_{32} = (-1)^5 \begin{vmatrix} 1 & -3 \\ 2 & 2 \end{vmatrix} = -1(2+6)$$

$$= -1(8) = -8$$

$$A_{33} = (-1)^6 \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 1(3-4)$$

$$= 1(-1) = -1$$

$$\therefore \text{ adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} -6 & 14 & -15 \\ 17 & 5 & 9 \\ 13 & -8 & -1 \end{bmatrix}^T$$

$$= \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$ **(1½)**

$$\therefore A^{-1} = \frac{1}{|A|} \cdot \text{adj}(A) = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

(1/2)

From Eq. (i), we get

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$ **(1)**

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67} \begin{bmatrix} 24+34+143 \\ -56+10-88 \\ 60+18-11 \end{bmatrix}$$

$$= \frac{1}{67} \begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

On comparing corresponding elements, we get

$\Rightarrow \quad x = 3, y = -2$ and $z = 1$ **(1)**

18. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

to solve the system of equations

$$x - y + 2z = 1$$
$$2y - 3z = 1$$

and $\quad 3x - 2y + 4z = 2$ **HOTS; Foreign 2011**

Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

Now, $AC = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

$$= \begin{bmatrix} -2-9+12 & 0-2+2 & 1+3-4 \\ 0+18-18 & 0+4-3 & 0-6+6 \\ -6-18+24 & 0-4+4 & 3+6-8 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ **(1½)**

$\Rightarrow \quad AC = I$

Now, on pre-multiplying both sides by A^{-1}, we get

$$A^{-1}AC = A^{-1}I$$

$\Rightarrow \quad IC = A^{-1}$

$[\because A^{-1}A = I$ and $A^{-1}I = A^{-1}]$

$\Rightarrow \quad C = A^{-1}$ **(1)**

$\therefore A^{-1} = C = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \end{bmatrix}$ **(1/2)**

$\lfloor 6 \quad 1 \quad -2 \rfloor$

Now, given system of equations can be written as

$$AX = B$$

where, $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 3 \\ 3 & -2 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

The solution of system of equation is given by

$$X = A^{-1}B$$

$\Rightarrow \quad X = CB \quad [\because A^{-1} = C]$ **(1)**

$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ **(1)**

$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2+0+2 \\ 9+2-6 \\ 6+1-4 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$

On comparing corresponding elements, we get

$x = 0, y = 5$ and $z = 3$ **(1)**

19. If $A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 0 & -1 \\ 2 & 6 & 0 \end{bmatrix}$, then find A^{-1}.

Using A^{-1}, solve the following system of equations

$$2x - y + z = -3$$
$$3x - z = 0$$
and $$2x + 6y - z = 2$$ **All India 2011C**

Given, $A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 0 & -1 \\ 2 & 6 & 0 \end{bmatrix}$

Now, $|A| = 2\,(0+6) + 1\,(0+2) + 1\,(18-0)$

$= 2(6) + 1(2) + 1(18)$

$$= 12 + 2 + 18 = 32$$

$\Rightarrow \quad |A| \neq 0$, hence unique solution exists. **(1)**

Now, cofactors of $|A|$ are

$$A_{11} = (-1)^2 \begin{vmatrix} 0 & -1 \\ 6 & 0 \end{vmatrix} = 1(0+6) = 6$$

$$A_{12} = (-1)^3 \begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix} = -1(0+2)$$

$$= -1(2) = -2$$

$$A_{13} = (-1)^4 \begin{vmatrix} 3 & 0 \\ 2 & 6 \end{vmatrix} = 1(18-0)$$

$$= 1(18) = 18$$

$$A_{21} = (-1)^3 \begin{vmatrix} -1 & 1 \\ 6 & 0 \end{vmatrix} = -1(0-6)$$

$$= -1(-6) = 6$$

$$A_{22} = (-1)^4 \begin{vmatrix} 2 & 1 \\ 2 & 0 \end{vmatrix} = 1(0-2) = -2$$

$$A_{23} = (-1)^5 \begin{vmatrix} 2 & -1 \\ 2 & 6 \end{vmatrix} = -1(12+2) = -14$$

$$A_{31} = (-1)^4 \begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix} = 1(1-0) = 1$$

$$A_{32} = (-1)^5 \begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} = -1(-2-3) = 5$$

$$A_{33} = (-1)^6 \begin{vmatrix} 2 & -1 \\ 3 & 0 \end{vmatrix} = 1(0+3) = 3$$

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 6 & -2 & 18 \\ 6 & -2 & -14 \\ 1 & 5 & 3 \end{bmatrix}^T \quad \textbf{(1½)}$$

$$= \begin{bmatrix} 6 & 6 & 1 \\ -2 & -2 & 5 \\ 18 & -14 & 3 \end{bmatrix} \quad \textbf{(1)}$$

$$\therefore \quad A^{-1} = \frac{1}{32}\begin{bmatrix} 6 & 6 & 1 \\ -2 & -2 & 5 \\ 18 & -14 & 3 \end{bmatrix} \quad \textbf{(1/2)}$$

Now, given system of equation can be written as $Ax = B$

where, $A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 0 & -1 \\ 2 & 6 & 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

and $\quad B = \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}$

The solution of system of equation is given by

$$X = A^{-1}B$$

$$\therefore \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{32}\begin{bmatrix} 6 & 6 & 1 \\ -2 & -2 & 5 \\ 18 & -14 & 3 \end{bmatrix}\begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix} \quad \textbf{(1)}$$

$$= \frac{1}{32}\begin{bmatrix} -18+0+2 \\ 6-0+10 \\ -54-0+6 \end{bmatrix} = \frac{1}{32}\begin{bmatrix} -16 \\ 16 \\ -48 \end{bmatrix} = \begin{bmatrix} -1/2 \\ 1/2 \\ -3/2 \end{bmatrix}$$

On comparing corresponding elements, we get

$$x = -\frac{1}{2}, y = \frac{1}{2} \text{ and } z = -\frac{3}{2} \quad \textbf{(1)}$$

20. If $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 2 & 0 & -3 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations

$$x - 2y + z = 0$$
$$-y + z = -2$$
and
$$2x - 3z = 10$$ **All India 2011C**

Given, $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 2 & 0 & -3 \end{bmatrix}$

Now, $|A| = 1(3 - 0) + 2\,(0 - 2) + 1(0 + 2)$
$= 1(3) + 2\,(-2) + 1(2) = 3 - 4 + 2 = 1$

$\Rightarrow |A| \neq 0$, hence unique solution exists. **(1)**

Now, cofactors of $|A|$ are

$$A_{11} = (-1)^2\,(3 - 0) = 1 \times 3 = 3$$
$$A_{12} = (-1)^3\,(0 - 2) = -1 \times -2 = 2$$
$$A_{13} = (-1)^4\,(0 + 2) = 1 \times 2 = 2$$
$$A_{21} = (-1)^3\,(6 - 0) = -1 \times 6 = -6$$
$$A_{22} = (-1)^4\,(-3 - 2) = 1 \times -5 = -5$$
$$A_{23} = (-1)^5\,(0 + 4) = -1 \times 4 = -4$$
$$A_{31} = (-1)^4\,(-2 + 1) = 1 \times -1 = -1$$
$$A_{32} = (-1)^5\,(1 - 0) = -1 \times 1 = -1$$
$$A_{33} = (-1)^6\,(-1 + 0) = 1 \times -1 = -1$$

$$\therefore \operatorname{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 3 & 2 & 2 \\ -6 & -5 & -4 \\ -1 & -1 & -1 \end{bmatrix}^T = \begin{bmatrix} 3 & -6 & -1 \\ 2 & -5 & -1 \\ 2 & -4 & -1 \end{bmatrix}$$ **(2½)**

$$\therefore A^{-1} = \begin{bmatrix} 3 & -6 & -1 \\ 2 & -5 & -1 \\ 2 & -4 & -1 \end{bmatrix}$$ **(1/2)**

[here, $|A| = 1$]

Now, given system of equations can be witten as

$$AX = B$$

where, $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 2 & 0 & -3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

and $B = \begin{bmatrix} 0 \\ -2 \\ 10 \end{bmatrix}$

whose solution is given by

$$X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 & -6 & -1 \\ 2 & -5 & -1 \\ 2 & -4 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ -2 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 0 + 12 - 10 \\ 0 + 10 - 10 \\ 0 + 8 - 10 \end{bmatrix}$$ **(1)**

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$$

On comparing corresponding elements, we get

$$x = 2, y = 0 \text{ and } z = -2$$ **(1)**

21. If $A=\begin{bmatrix}1 & -2 & 0\\ 2 & 1 & 3\\ 0 & -2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}7 & 2 & -6\\ -2 & 1 & -3\\ -4 & 2 & 5\end{bmatrix}$,

then find AB and hence solve system of equations

$$x-2y=10$$

$$2x+y+3z=8$$

and $$-2y+z=7$$ **Delhi 2011C**

Given,

$$A=\begin{bmatrix}1 & -2 & 0\\ 2 & 1 & 3\\ 0 & -2 & 1\end{bmatrix} \text{ and } B=\begin{bmatrix}7 & 2 & -6\\ -2 & 1 & -3\\ -4 & 2 & 5\end{bmatrix}$$

Firstly, we find product AB and then use it to find inverse A^{-1}. **(1)**

Now, $$AB=\begin{bmatrix}1 & -2 & 0\\ 2 & 1 & 3\\ 0 & -2 & 1\end{bmatrix}\begin{bmatrix}7 & 2 & -6\\ -2 & 1 & -3\\ -4 & 2 & 5\end{bmatrix}$$

$$=\begin{bmatrix}7+4-0 & 2-2+0 & -6+6+0\\ 14-2-12 & 4+1+6 & -12-3+15\\ 0+4-4 & 0-2+2 & 0+6+5\end{bmatrix}$$

$$\Rightarrow \quad AB=\begin{bmatrix}11 & 0 & 0\\ 0 & 11 & 0\\ 0 & 0 & 11\end{bmatrix}=11I$$ **(1½)**

$$\Rightarrow \quad AB=11I$$

On pre-multiplying both sides of Eq. (i) by A^{-1}, we get

$$A^{-1}AB=11A^{-1}I \Rightarrow IB=11A^{-1}$$

$$[\because A^{-1}A=I \text{ and } A^{-1}I=A^{-1}]$$

$$\Rightarrow \quad B=11A^{-1} \qquad [\because IB=B]$$

$$\Rightarrow \quad A^{-1}=\frac{1}{11}\cdot B$$

$$\Rightarrow \quad A^{-1}=\frac{1}{11}\begin{bmatrix}7 & 2 & -6\\ -2 & 1 & -3\\ \end{bmatrix}$$ **(1½)**

$$\frac{1}{11}\begin{bmatrix} & & \\ -4 & 2 & 5\end{bmatrix}$$

Now, given system of equations can be written as $AX = B$

where, $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

whose solution is given by $X = A^{-1}B$.

$$\Rightarrow X = \frac{1}{11}\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} \quad \textbf{(1)}$$

$$= \frac{1}{11}\begin{bmatrix} 70 + 16 - 42 \\ -20 + 8 - 21 \\ -40 + 16 + 35 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 44 \\ -33 \\ 11 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$$

On comparing corresponding elements, we get

$x = 4, y = -3$ and $z = 1$. **(1)**

22. If $A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations

$$3x - 4y + 2z = -1$$
$$2x + 3y + 5z = 7$$

and $$x + z = 2$$ **Delhi 2011C**

Given, $A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$

Now, $|A| = 3(3 - 0) + 4(2 - 5) + 2(0 - 3)$

$= (3 \times 3) + (-4 \times 3) + (2 \times -3)$

$= 9 - 12 - 6 = 9 - 18 = -9$

$\Rightarrow |A| \neq 0$, hence unique solution exists. **(1)**

Now, cofactors of $|A|$ are

$A_{11} = (-1)^2 (3 - 0) = 1 \times 3 = 3$

$$A_{12} = (-1)^3\,(2-5) = -1 \times -3 = 3$$

$$A_{13} = (-1)^4\,(0-3) = 1 \times -3 = -3$$

$$A_{21} = (-1)^3\,(-4-0) = -1 \times -4 = 4$$

$$A_{22} = (-1)^4\,(3-2) = 1 \times 1 = 1$$

$$A_{23} = (-1)^5\,(0+4) = -1 \times 4 = -4$$

$$A_{31} = (-1)^4\,(-20-6) = 1 \times -26 = -26$$

$$A_{32} = (-1)^5\,(15-4) = -1 \times 11 = -11$$

$$A_{33} = (-1)^6\,(9+8) = 1 \times 17 = 17$$

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 3 & 3 & -3 \\ 4 & 1 & -4 \\ -26 & -11 & 17 \end{bmatrix}^T \qquad \textbf{(1½)}$$

$$= \begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix} \qquad \textbf{(1)}$$

$$A^{-1} = -\frac{1}{9}\begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix} \qquad \textbf{(1/2)}$$

Now, given system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$$

whose solution is given by $X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{9}\begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix}\begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix} \qquad \textbf{(1)}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{9}\begin{bmatrix} -3+28-52 \\ -3+7-22 \\ 3-28+34 \end{bmatrix} = -\frac{1}{9}\begin{bmatrix} -27 \\ -18 \\ 9 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

On comparing corresponding elements, we get

$$x = 3, y = 2 \text{ and } z = -1. \quad \textbf{(1)}$$

23. If $A = \begin{bmatrix} 8 & -4 & 1 \\ 10 & 0 & 6 \\ 8 & 1 & 6 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations

$$8x - 4y + z = 5$$
$$10x + 6z = 4$$
and $$8x + y + 6z = \frac{5}{2}$$ **All India 2010C**

Do same as Que. 22.

$$\left[\textbf{Ans. } A^{-1} = \frac{1}{10}\begin{bmatrix} -6 & 25 & -24 \\ -12 & 40 & -38 \\ 10 & -40 & 40 \end{bmatrix} \text{ and } x = 1, y = \frac{1}{2}, z = -1\right]$$

24. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$,

then find AB. Use this to solve the system of equations

$$x - y = 3$$
$$2x + 3y + 4z = 17$$

and $$y + 2z = 7$$ **All India 2010C**

Do same as Que. 21.

[Ans. $AB = 6I$ and $x = 2, y = -1, z = 4$]

25. If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, then find A^{-1}. Hence, solve the following system of equations

$$3x + 2y + z = 6$$
$$4x - y + 2z = 5$$

and $$7x + 3y - 3z = 7$$ **Delhi 2010C**

Do same as Que. 22.

$$\left[\begin{array}{l} \textbf{Ans. } A^{-1} = \frac{1}{62}\begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 5 & 5 & -11 \end{bmatrix} \\ \text{and } x = 1, y = 1, z = 1 \end{array}\right]$$

26. Using matrices, solve system of linear equations

$$x + y + z = 6$$
$$x + 2z = 7$$

and $$3x + y + z = 12$$ **All India 2009**

Do same as Que. 17.

[Ans. $x = 3, y = 1$ and $z = 2$]

27. Solve the following system of equations, by using matrix method.

$$3x - 2y + 3z = 8$$
$$2x + y - z = 1$$
and $$4x - 3y + 2z = 4$$ **Foreign 2009**

Do same as Que. 17.

[**Ans.** $x = 1, y = 2$ and $z = 3$]

28. Using matrices, solve the following system of equations

$$x + y + z = 1$$
$$x - 2y + 3z = 2$$
and $$x - 3y + 5z = 3$$ **All India 2009C**

Do same as Que. 17.

$\left[\textbf{Ans. } x = \frac{1}{2}, y = 0 \text{ and } z = \frac{1}{2}\right]$.

29. Using matrices, solve the following system of equations

$$8x + 4y + 3z = 18$$
$$2x + y + z = 5$$
and $$x + 2y + z = 5$$

All India 2009C

Do same as Que. 17.

[**Ans.** $x = 1, y = 1$ and $z = 2$]

30. Using matrices, solve the following system of equations

$$x + y - z = 3$$
$$2x + 3y + z = 10$$
and $$3x - y - 7z = 1$$ **Delhi 2009C**

Do same as Que. 17.

[**Ans.** $x = 3, y = 1$ and $z = 1$]

31. Using matrices, solve the following system of equations

$$2x + 8y + 5z = 5$$
$$x + y + z = -2$$
and $$x + 2y - z = 2$$ **Delhi 2009C**

Do same as Que. 17.

[**Ans.** $x = -3, y = 2$ and $z = -1$]

32. Using matrices, solve the following system of equations

$$x - y + 2z = 1$$
$$2y - 3z = 1$$
and $$3x - 2y + 4z = 2$$ **All India 2008C**

Do same as Que. 17.

[**Ans.** $x = -3, y = 2$ and $z = -1$]

33. Using matrices, solve the following system of equations

$$2x + y + z = 7$$
$$x - y - z = -4$$
and $$3x + 2y + z = 10$$ **All India 2008C**

Do same as Que. 17.

[**Ans.** $x = 1, y = 2$ and $z = 3$]